Honors Analysis Chapter 4

"Mechanical"

2003

				Exam Chapter 3 3 / 28
Important Lecture # 1 Part I A, B, C **Do even numbered problems ONLY Product & Quotient rules** 3 / 31	**Industrial Strength Derivatives** # 2 Part I A, B, C **Do odd numbered problems ONLY** 4 / 1	**Important Lecture** # 3, 4 ISD section II section II Part A & B 4 / 2	~~Minimum Day~~ # 5 Unchained Melody **Chain Rule** 4 / 3	~~Minimum Day~~ **Important Lecture** # 6 ISD section III **Implicit Derivatives** 4 / 4
# 7 **ISD IV V problems a - i** **Trig functions** 4/ 7	# 8 **ISD V problems j - r** 4/ 8	# 9 **ISD VI, VII a,b** 4/ 9	# 10, 11 **VII c, d, e, f** 4/ 10	Begin Spring Break work Due 4/21 #12, 13, 14 "Newton Searches for Easter Eggs" 4/ 11
Spring Break	**also during break, *BEGIN***		**# 15, 16, 17, 18, 19 "Heston's Derivative Marathon"**	
¿Did somebody parlez el Quizzo?	une autra la quizzette????		Put Last Year's Exam on Chalkboards	**Exam Chapter 4 "Mechanical" 2003**
"Heston's Derivative Marathon" is due end of period 4/23				
4/ 21	4/ 22	4/ 23	4/ 24	4/ 25

Now you can take the derivative of almost anything. In the next unit we shall find out what we can do with these derivatives. Does it sound like we're heading toward devine word problems? It should. We love Analysis!!

Industrial Strength Derivatives

Mr. Mumaugh

I A **For each of the following, find the first derivative two times. First use the product rule. Next, clear parentheses in the original expression and simplify. Then, take the derivative in the usual manner. Reconcile any differences in final answers.**

1) $8x(3x+2)$ 2) $2x^3\left(x-x^3\right)$ 3) $\left(3x^2+2x\right)(5x-2)$ 4) $3x^3(4x+8)$

5) $3x^2\left(x-\sqrt{x}\right)$ 6) $x^2\left(x^2+x^2\right)$ 7) $(3x+3)(3x-3)$ 8) $\left(2-3x^2\right)\left(x^2+2\sqrt{x}\right)$

9) $(2x+1)(1-2x)$ 10) $(2x+3)^3$ 11) $(2x+3)^4$ 12) $(2-3x)^3$

B Use the quotient rule to find the derivative:

1) $\dfrac{(2x-1)}{3x-2}$ 2) $\dfrac{(2x-1)^2}{3x^2-2}$ 3) $\dfrac{(2x-1)}{3x^2}$ 4) $\dfrac{2x-3}{x}$

5) $\dfrac{ax+d}{c-dx}$ 6) $\dfrac{3x-5}{5-3x}$ 7) $\dfrac{3x+2}{8-12x}$ 8) $\dfrac{5x^2+2}{2x^3-5}$

C Find the derivative:

1) $\dfrac{3x(2x-1)}{5x+2}$ 2) $\dfrac{(3x+2)(2x-3)}{(5x+1)}$ 3) $\dfrac{(3x+2)(2x-3)}{(3-2x)(5x+1)}$ 4) $\dfrac{(x+2)(3x-4)}{(x-2)(5x+1)}$

II A **Find the derivative twice. First clear parentheses and use the "quick derivative." Second, use the chain rule. Clear parentheses to verify that you got the same derivative.:**

1) $(3x-2)^2$ 2) $\left(3\sqrt{x}-2\right)^2$ 3) $\left(2x^2+4x+1\right)^2$ 4) $(2x+1)^4$

B Use the chain rule to find the derivative. Note that some problems may require a combination of the product rule, quotient rule as well as the chain rule. Be sure that your work makes it perfectly clear what method you used! Show the "u"s and the "v"s.

1) $(3x-1)^2(5-2x)^3$ 2) $\dfrac{\left(1-3x^2\right)^3}{(4x)^2}$ 3) $\dfrac{\left(x^2+1\right)^3}{3x-1}$ 4) $\dfrac{1}{(2x+3)^2}$

5) $\dfrac{1}{(2x+3)}$ 6) $x\left(\dfrac{x^2}{x+1}\right)$ 7) $\left(\dfrac{1-2x}{2+x}\right)^2$ 8) $\dfrac{(1-2x)^2}{(2+x)^2}$

9) $\sqrt{4x^2+2x+1}$ 10) $\sqrt{5-3x^2}$ 11) $\dfrac{1}{\sqrt{5-3x^2}}$ 12) $\dfrac{3x}{\sqrt{1-x^2}}$

13) $\dfrac{(x+1)^2(x-1)^2}{\left(x^2-1\right)^2}$ 14) $x(x-1)\left(x^2-x+1\right)$ 15) $\dfrac{x^2+1}{x^2}$ 16) $1+\dfrac{1}{x^2}$

17) $\sqrt{\dfrac{2x-5}{3x-8}}$ 18) $(2+3)^{17}$ 19) $\dfrac{x^2-1}{x-1}$ 20) $\dfrac{x^3-1}{x-1}$

21) $\left((2x-1)^2+2x\right)^2$ 22) $\left(4x^2-2x+1\right)^2$ 23) $16x^4-16x^3+9x^2-8x+1$

more Industrial Strength Derivatives

III Find $\frac{dy}{dx}$

a) $x^2 + y^2 = 16$

b) $x^2+y^2-5x-8y-14=0$

c) $xy^2 = x+y$

d) $x^{2/3} + y^{2/3} = 8^{2/3}$

e) $x^2 - y^2 = 1$

f) $x^2 - 2xy + 4y^2 - 4x = 0$

g) $y^2 = \frac{x^2}{x^2 + 4}$

h) $5x^2 + 4xy + 3y^2 + y = 0$

i) $(x+y)^2+(x-y)^2=16+2y$

j) $4xy+x^2+2y^2=25$

k) $x^2 = \frac{x-y^2}{x+y}$

l) $\sqrt{x+y}+\sqrt{x+2y}=10$

IV There are six standard trig functions. What are their derivatives? (hint: $\sec x = \frac{1}{\cos x} = (\cos x)^{-1}$)

V Find $f'(x)$ two times. First take the derivative of the given as it stands. Second, change the given using a trig identity and then take the derivative. Evaluate both of your answers at $x = \frac{\pi}{4}$.

a) $f(x) = 2 \sin 2x \cos 2x$

b) $f(x) = \frac{1}{2} \sin 4x$

c) $f(x) = \frac{\tan^4 x}{\sec^2 x}$

d) $f(x) = \sin^2 2x + \cos^2 2x$

e) $f(x) = \frac{\sin 2x}{\cos x}$

f) $f(x) = (\sin^2 x + \cos^2 x)$

g) $f(x) = 1 - \sin^2 x$

h) $f(x) = \sin 2x$

i) $f(x) = 2 \sin x \cos x$

j) $f(x) = \frac{1}{2}(1 + \cos 2x)$

k) $f(x) = \frac{1}{2}(1 - \cos 2x)$

l) $f(x) = (\sin x + \cos x)^2$

m) $f(x) = \frac{\cos^2 x}{1 - \sin x}$

n) $f(x) = \sin x \tan x + \cos x$

o) $f(x) = \frac{1 + 2 \sin x \cos x}{\sin x + \cos x}$

p) $f(x) = \frac{1 - \cos 2x}{\sin 2x}$

q) $f(x) = \sec^2 x$

r) $f(x) = \frac{2 \tan x}{1 + \tan^2 x}$

VI Find $f'(x)$. Do not simplify. Be sure to do all the pieces in the proper order.

a) $f(x) = \left(\frac{\sqrt{3x^2 - x}}{5x^3 + 1}\right)^{\frac{2}{3}}$

b) $f(x) = \frac{(3x^3 + 2)(2x - 3)^2}{\sqrt{5x^3 + 1}}$

c) $f(x) = \frac{5(\sin^2 x - \cos x)^3}{6\sqrt{\sin x \cos x}}$

VII Prepare proper numberlines for each of the following. Sketch a graph of f. Identify extrema and PI. Justify as usual. The stated intervals are "windows of view" and not necessarily the valid domains.of the functions.

a) $f(x) = \sin x + x \; ; \; -2\pi \le x \le 2\pi$

b) $f(x) = \sin x + \cos x \; ; \; -2\pi \le x \le 2\pi$

c) $f(x) = \frac{\cos x}{1 + \sin x} \; ; \; -2\pi \le x \le 2\pi$

d) $f(x) = \sin\left(\frac{x^2}{1 + x^2}\right) \; ; \; -2\pi \le x \le 2\pi$

e) $f(x) = \sqrt{x - x^2} \; ; \; -2 \le x \le 2$

f) $f(x) = x^6 - x^4 \; ; \; -2 \le x \le 2$

Clue: Newton's Method of finding roots involves the x-intercept of the tangent line.

Consider: Let g = your first guess at a root to an equation or a zero of a function.
The coordinate of interest is $\big(g, f(g)\big)$ and the slope of the tangent line at that point is $f'(g)$.
Write the equation of the line; solve for x; this is your next guess.
Start over with this latest value of x. When f (of your last x choice) = 0, you have the root!

Clue: Page 173 ...

For problems 1 and 2, find guess number 3 for each problem.

If you wanted to find an approximate value for $\sqrt{3}$ you would Let $x = \sqrt{3}$. so $x^2 = 3$ It follows that $x^2 - 3 = 0$. Now we want a zero of our equation or we could phrase it as $f(x) = x^2 - 3$. This way we want a zero for our equation. (since $f\big(\sqrt{3}\big) = 0$).

Now you want to try a guess like $x = 2$. Find $\big(2, f(2)\big)$ and $f'(2)$ etc. Use your calculator when the numbers get too messy.

the law of motion of a particular particle is given by:

Lecture #2
ch 4 5 6 mech

$s = f(t) = 8t - t^2$ this describes the position of a particle at some time t. "Position" should be interpretted as the particle's distance from some given point. Usually this given point is where the particle is at $t = 0$. In keeping with normal practice, we say if the particle is to the right of a given point we say it is a +. If the particle is to the left of a given pt we say it is −. E.g. if $s = -6$ we say the particle is 6 units to the left of "Home". etc.

① Let's determine how fast the particle is moving between the time 2 and 3 seconds.

$f(t) = -1 \quad 8 \quad 0$

$\frac{f(3) - f(2)}{3 - 2} = \frac{15 - 12}{3 - 2} = 3$

2	−1	+6	12
3	−1	5	15
2.5	−1	5.5	13.75
2.1	−1	5.9	12.39

Thus the AVERAGE velocity is 3 units/sec.

② Lets determine velocity when for the interval 2 to 2.5

$$\frac{f(2.5) - f(2)}{2.5 - 2} = \frac{13.75 - 12}{.5} = \frac{1.75}{.5} = 3.5$$

③ Determine velocity between 2 and 2.1

$$\frac{f(2.1) - f(2)}{2.1 - 2} = \frac{12.39 - 12}{.1} = \frac{.39}{.1} = 3.9$$

④ Generally $\frac{f(t + \Delta t) - f(t)}{(t + \Delta t) - t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$ is the Average Velocity.

⑥ what would this represent?:

$$\lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$ instantaneous velocity

Have you ever seen it done like this? Look at what would this represent? old duffo sheets.

④ $\lim_{z \to 2} \frac{f(z) - f(2)}{z - 2}$ the instantaneous velocity at $t = 2$.

turn to page 122, what have you marked with a box on that page?

this instantaneous velocity at a point becomes simply f' at that pt.

✳ Notice: I actually do not care EXACTLY where the particle is. Simply how far it is from "home". Compare this idea to that of drawing the graph of f' given the curve of f. I did not care exactly where the curve went so long as it was negative when f was decreasing and positive when f was increasing.

We say then the velocity of the particle is the derivative of the fctn. That is, the derivative of the distance, s, with respect to time, t, is the velocity. ie

$$v = \frac{ds}{dt} = f'(t).$$

Consider eg 1 pg 164.

$S = f(t) = t^3 - 6t^2 + 9t \quad t \geq 0$

a) find Velocity of P at time t $\quad v = 3t^2 - 12t + 9$

b) find acceleration of P at time t $\quad a = 6t - 12$

c) find coordinates of P when P is at rest
ie when velocity is 0.

$v = 3t^2 - 12t + 9 \overset{\text{set}}{=} 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3 \quad t = 1$

$f(1) = 4 \Rightarrow (1, 4)$

$f(3) = 0 \Rightarrow (3, 0)$

f ⇒	1	-6	9	0
1 ⌋	1	-5	4	0
3 ⌋	1	-3	0	0

skip the graph similar to the eg.

d)

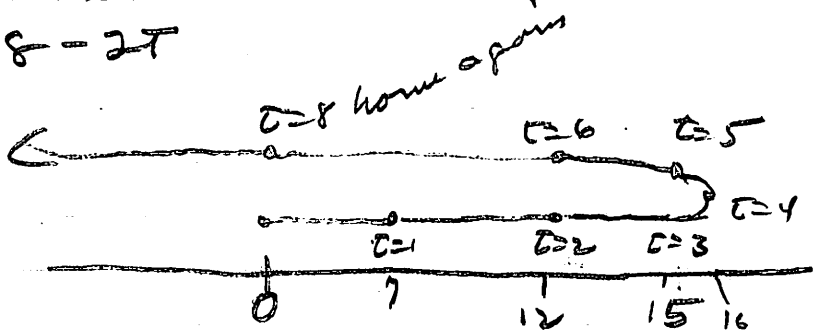
we have been discussing a fctn that applies to time and distance, actually we began with this because the idea of an object traveling is easy to visualize, actualy the theory I used applies for any fctn.

Let's ANALize $f(t)$ a bit more.

$f(t) = s = 8t - t^2$

$f'(t) = 8 - 2t$

24
−9
15



That is at $t=1$ the particle is 7 units from home.

at $t=2$ it is 12 units from home.
at $t=3$ " " 15 " " " "
at $t=4$ " " 16 " " " "
at $t=5$ " " 12 " " " "

Notice the instantious Velocity (Velocity) I will state average velocity if I am considering it)

at $t=2$ $V=4$ units per sec.
$t=3$ $V=2$ units per sec.
$t=4$ $V=0$ Stopped,
$t=5$ $V=-2$ Moving left at 2 u/sec
$t=8$ $V=-8$

We Noticed that the particle seems to be slowing down to a stop then increase in velocity. This appears to be a deceleration then an acceleration. Or a change in Velocity. Notice $\frac{ds}{dt} = f'$ was a change in distance with respect to time. Thus a change in velocity with respect to

time would be $\frac{d(f(t))}{dt} = f''(t)$.

We say the second derivative is the acceleration of the particle. ~~Obviously if $f'' > 0$ it is increasing in velocity and if $f'' < 0$ it is decreasing in velocity, again if $f'' = 0$~~

Normally we say a particle slows down or speeds up. Thus we say a negative acceleration is deceleration and the velocity is less at each time t. However, the deceleration may slow while the speed increases.

Not possible? Two definitions are required.

(1) Velocity is a directed speed. a particle moves to the right at 5 miles per second we say the velocity is $+5$. If the particle moves to the left at 5 miles per second we say the velocity is -5

(2) Speed is the magnitude of, or the absolute value of, the velocity.

The Speedometer measures instantaneous speed. to determine average speed you divide distance traveled by the time required to travel that distance.

Instantaneous

Summary after Students 1st Attempt at Pg 166-169.

164-168

Summary: Given $f(\tau)$ then we can find the velocity of P at any time a simply by finding $f'(a)$.

we know that P moves along $f(\tau)$. at $\tau = 2$ P is traveling at 4 ft/sec. we found that at $\tau = 3$ P is traveling at 2 ft/sec. we notice that P is actually slowing down.

Question. at what rate is P slowing?

Since $f'(\tau)$ is the rate of change for P following f. $f''(\tau)$ is the rate of change of the velocity of P.

$f(\tau) = 8\tau - \tau^2$ $f'(\tau) = 8 - 2\tau$ $f''(\tau) = -2$
ie P is slowing down at a rate of 2 feet per second. ~~P is decelerating at the rate of 2 ft/sec.~~

Question what happens to P at $\tau = 4$?
It is momentarily stopped.

Question what happens at $\tau = 5$

$f(5) = 15$ $f'(5) = -2$ $f''(5) = -2$

at $\tau = 5$ P is at (5, 15) is is traveling at the rate of 2 feet/sec and traveling Down. (Since one cannot travel at a Negative speed, the sign is interrupted as being the direction of travel.

$f''(\tau)$ is called the acceleration of P. whether P is actually slowing down or speeding up.

Summary

164-168

End of Lecture #2 Ch 4 5 & much

an object that moves has a path. If that object will always follow a certain path we can write a formula describing that path.

We have such an equation $s = f(t) = 8t - t^2$ the object P moves along this path. where s is the position of P at time t. s is measured in feet & t measured in seconds.

Question. In 2 seconds where is P?

$f(2) = 12$ ie $(2, 12)$

Question. How fast is it moving at that point? rephrased: P moves from one point to another at some rate. what is that rate.

we could guess that the rate or speed is the same at 2 as it is between 2 and 3 sec. Thus $f(3) - f(2)$ per second. $15 - 12 = 3$ feet/sec. Is that the rate at time 2 or the average rate between the rates of $f(2)$ and $f(3)$?

average.

In order to find the exact rate of change at time = 2 we could pick closer and closer approximations to time = 2 or simply take the limit. This becomes however the derivative of f evaluated at 2 ie $f'(2)$.

Since $f(x) = 8t - t^2$ $f'(x) = 8 - 2t$

Thus $f'(2) = 8 - 4 = 4$ Answer at the instant that $t = 2$, P is traveling at the rate of 4' per second.

at $f'(3) = 8 - 6 = 2$ ie Traveling at the rate of only 2 feet per second!

we say the rate an object follows its path is that object's velocity.

173-177
lecture #1 #1
Ch 4.5.6
mech

~~Question~~ Given $f(x) = x^3$
$g(x) = x^2$

Question $\frac{d(f(x)+g(x))}{dx} \stackrel{?}{=} f'(x) + g'(x)$

Yes

Question $\frac{d(f(x)g(x))}{dx} \stackrel{?}{=} (f'(x))(g'(x))$

NO.

thm. If f and g are both differentiable at x, then fg is diff at x and is equal to:

Product rule. $(fg)'(x) = f(x)g'(x) + g(x)f'(x)$
$(uv)'(x) = u\,dv + v\,du$

it "the product of the derivative is equal to the 1st times derivative of 2nd plus 2nd times derivative of the 1st"

Discuss eg 1 pg 175

Eg 2 pg 175

given: $f(x) = \sqrt{x}$ then $f'(x) = \frac{1}{2\sqrt{x}}$
find $\phi'(x)$ if $\phi(x) = \sqrt{x}(x^2+1)$ for $x > 0$

let $f(x) = \sqrt{x}$ and $g(x) = (x^2+1)$
$\phi'(x) = f(x)g(x) = \frac{1}{2\sqrt{x}}(x^2+1) + 2x(\sqrt{x})$
$\frac{x^2+1}{2\sqrt{x}} + 2x\sqrt{x} = \frac{x^2+1+4x^2}{2\sqrt{x}} = \frac{5x^2+1}{2\sqrt{x}}$

Do Demo Problems.

$u \quad v$

$(3x+1)(5x-2)$

$v\,du + u\,dv$ ← include this

$(5x-2)3 + (3x+1)5$

$15x - 6 + 15x + 5$

$30x - 1$

$15x^2 - x - 2$

$30x - 1$

END of lecture #3 Ch 4 5 6 as we reach

If f and g are both differentiable at x and $g(x) \neq 0$, then $\frac{f}{g}$ is diff at x and

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\left(\frac{u}{v}\right)' = \frac{v\,du - u\,dv}{v^2}$$

Eg 3 pg 175

a) $\phi(x) = \frac{x^3}{x^2+1}$ let $f(x) = x^3$ let $g(x) = x^2+1$

$$\frac{3x^2(x^2+1) - x^3(2x+0)}{(x^2+1)^2}$$

$$\frac{3x^4+3x^2-2x^4}{(x^2+1)^2} = \frac{x^4+3x^2}{(x^2+1)^2}$$

finally, show

$\sqrt{x} = x^{\frac{1}{2}} \Rightarrow (\sqrt{x})' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

Eg 3 b) $\phi(x)$

EMU

① $y = x^2(x^2+1)$

$f = x^2$ $\quad f' = 2x$ $\quad g = x^2+1$ $\quad g' = 2x$ $\quad 2x^3 + 2x^3 + 2x =$

$4x^3 + 2x = y'$

② $y = (x+2)(x^2+1)$

$f = x+2$ $\quad f' = 1$ $\quad g = x^2+1$ $\quad g' = 2x$ $\quad x^2+1 + 2x^2 + 4x$

$3x^2 + 4x + 1 = y'$

③ $y = (x^2+1)(x^2+x)$

$f = x^2+1$ $\quad f' = 2x$ $\quad g(x) = x^2+x$ $\quad g'(x) = 2x+1$ $\quad 2x^3 + 2x^2 + (x^2+1)(2x+1)$

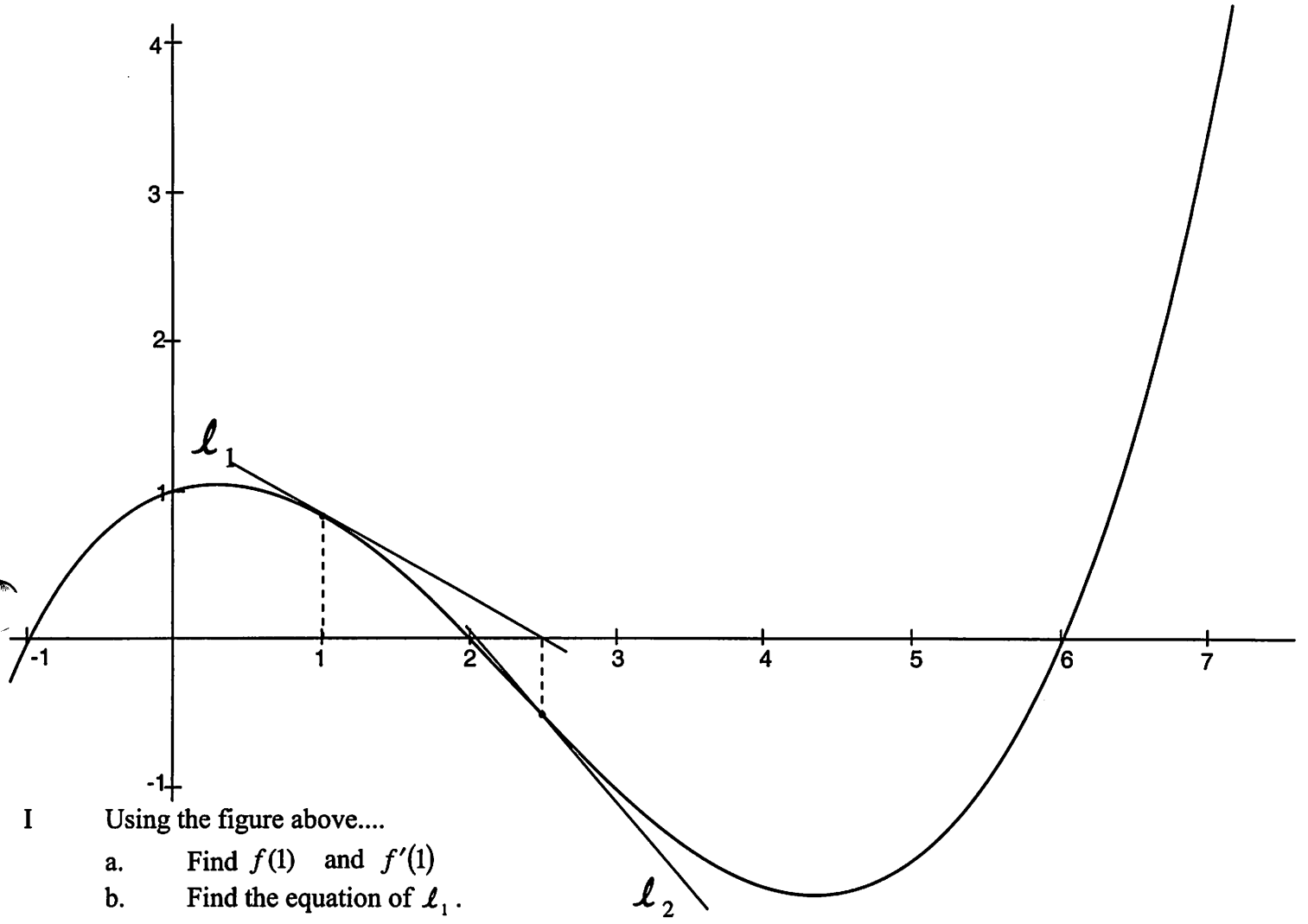
$2x^3 + 2x^2 + 2x^3 + x^2 + 2x + 1$

$4x^3 + 3x^2 + 2x + 1$

Honors Analysis **Newton Searches for Easter Eggs** Name_____

Mr. Mumaugh Per_____

Given: $f(x)=\left(\frac{1}{12}\right)(x^3-7x^2+4x+12)$



I Using the figure above....

a. Find $f(1)$ and $f'(1)$

b. Find the equation of ℓ_1.

c. Find the x-intercept of ℓ_1.

d. The figure has a dotted line from your answer to part c to the graph of f. Using your calculator, find $f(2.4286)$ and $f'(2.4286)$.

e. Compute the x-intercept of ℓ_2. That is the third guess. (our first guess was 1, our second guess was 2.4286) _____

f. Compute the 4th guess._____

II Do the same thing as we did in section I while seeking the root at $x=6$. Use $x=5$ as your first guess. Draw the lines (for 1st, 2nd, and 3rd guesses). Label them including the x-intercepts. Fill in the table:

1^{st} *guess* $\rightarrow$ 5	4^{th} *guess* $\rightarrow$ _____
2^{nd} *guess* $\rightarrow$ _____	5^{th} *guess* $\rightarrow$ _____
3^{rd} *guess* $\rightarrow$ _____	6^{th} *guess* $\rightarrow$ _____

Honors Analysis

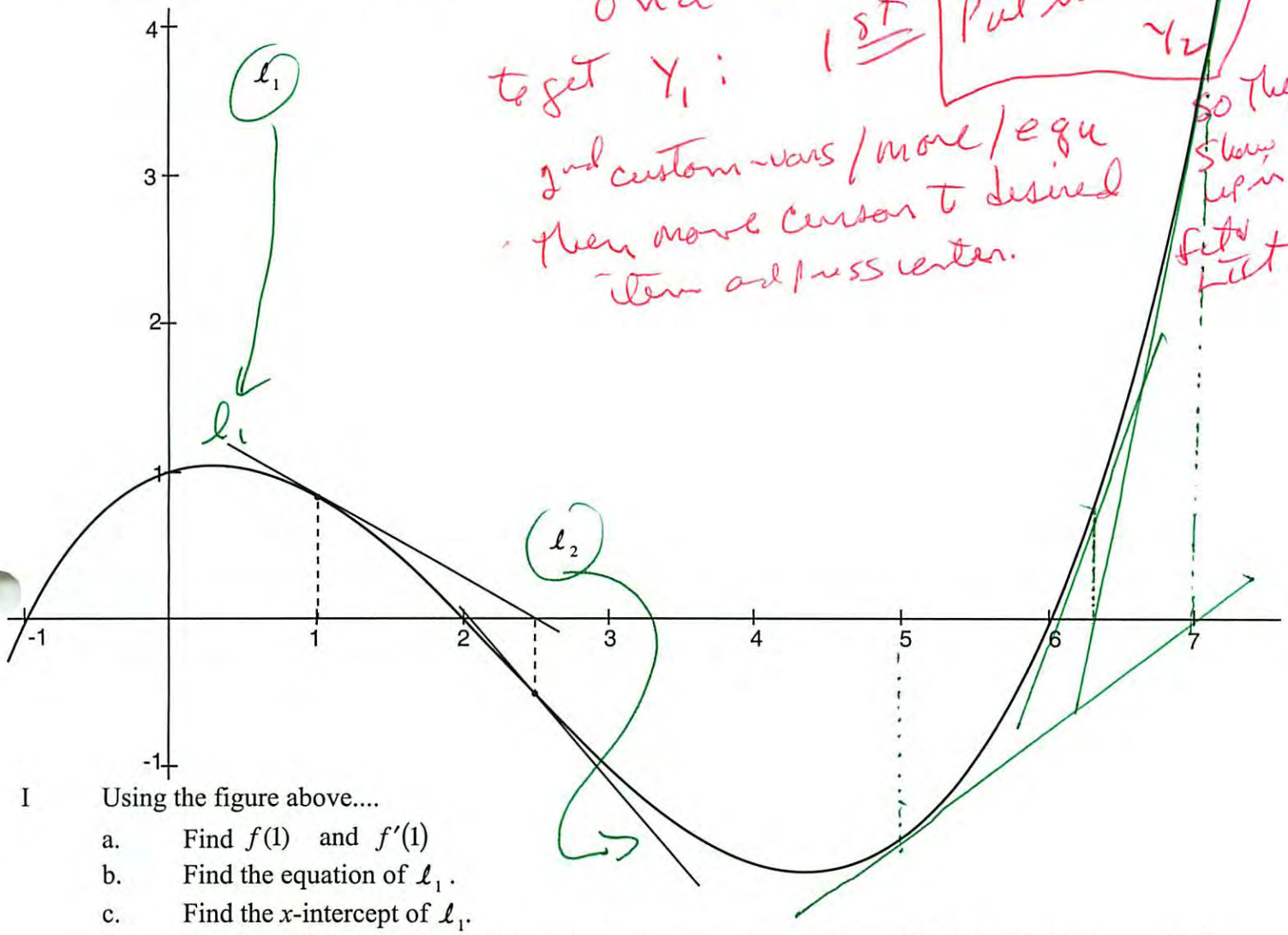
Newton Searches for Easter Eggs

Name _____

Mr. Mumaugh

Per _____

Given: $f(x) = \left(\frac{1}{12}\right)(x^3 - 7x^2 + 4x + 12)$



I Using the figure above....

a. Find $f(1)$ and $f'(1)$

b. Find the equation of ℓ_1.

c. Find the x-intercept of ℓ_1.

d. The figure has a dotted line from your answer to part c to the graph of f. Using your calculator, find $f(2.4286)$ and $f'(2.4286)$.

e. Compute the x-intercept of ℓ_2. That is the third guess. (our first guess was 1, our second guess was 2.4286) 2.0021

f. Compute the 4th guess. 2.00000038

II Do the same thing as we did in section I while seeking the root at $x = 6$. Use $x = 5$ as your first guess. Draw the lines (for 1st ~~and~~ 2nd guess). Label them including the x-intercepts. Fill in the table:

1st *guess* →	5	4th *guess* →	6.0206
2nd *guess* →	7	5th *guess* →	6.00016
3rd *guess* →	6.245	6th *guess* →	6.000000001

Let $f(x)$ be the function for which we are seeking the x-intercept. (ie That would be the zero of the function). It follows that $f'(x)$ would be the derivative of f. (Otherwise known as the slope of the tangent line to the function at the value x.)

Let the first guess be x_1.

The coordinate $(x_1, f(x_1))$ is the point on the curve and the tangent line at that point has slope $f'(x)$.

Write the equation of the tangent line using $y - y_1 = m(x - x_1)$. Solve for y.

$y - f(x_1) = f'(x_1)(x - x_1)$

$y = f'(x_1)x + f(x_1) - x_1 f'(x_1)$

As in the Section I and II, we want the x-intercept and that is when the y values is zero .... so ....
we let $y = 0$ and get:

$$0 - f(x_1) = f'(x_1)(x - x_1).$$

Solve for x ↑ Remember that x_1 is **NOT** a variable. It was our first guess.

$0 = f'(x_1)x + f(x_1) - x_1 f'(x_1)$

$f'(x_1)x = x_1 f'(x_1) - f(x_1)$

$x = x_1 - \frac{f(x_1)}{f'(x_1)}$

III Let's check the work for the first two sections using the calculator. We will program the calculator with the result you found above and check for accuracy.

We need to tell the computer the function we are working with so locate the Y= key and press.
Make $Y_1 =$ read: $Y_1 = (1/12)(x\text{^}3 - 7x^2 + 4x + 12)$
Make $Y_2 =$ read: $Y_2 = (1/12)(3x^2 - 14x + 4)$ (NOTE: We could have made the calculator compute the derivative for us.... in that case we would make $Y_2 =$ read: $Y_2 = nDer(Y_1, X, X)$. nDer(is found under the Math menu. Y_1 is found in the catalog or in Vars or in Y-Vars. The first X tells the calculator that the variable is x. and the second X tells the calculator to check the variable X for the value to evaluate the function at each time it computes it (rather than to only compute the value one time---- at a specific point).

On the next page, we will begin the actual program. Each line you are supposed to enter into the program will have enter this line → immediately in front of the line you are supposed to enter. The other text will be explaining what is being done and/or where to find the commands.

Turn the calculator on and press the "program" key. Use the arrow keys to move to NEW. Press Enter.
The calculator needs to know what to call the program.

enter this line → NEWTON

Press Enter.
We want the "program" to ask us for the 1st x-value to start so under the I/O menu select

enter this line → INPUT X

Note that X is the variable X notated as x-var on the calculator.
We will want the program to use what ever value it gets as the next guess so we will want the program to return to this same spot to compute the value again so we tell it this is the spot by putting a Label here:

enter this line → Lbl 1

Now we key in the formula you found on page 2.

enter this line → $X - (Y_1(X))/Y_2(X) \rightarrow X$

This says, from the value we put in as our guess subtract the fraction computed as $f(x)$ divided by $f'(x)$ and put that new value back in the variable X.
We want to see the value the calculator found so we Display it....

enter this line → Disp X

The program will happen very quickly so we could never see the results unless we tell the machine to wait until we have see the value and specifically told it to carry on.

enter this line → Pause

After we have seen the value we want that value to be used on the next computation so we want the program to return to the spot we earlier labeled as number 1

enter this line → Goto 1

Press 2nd Quit to exit the program mode.

Every time we press Enter we will get a new x value. It is surprising how quickly that value approaches zero.

To stop the program from running press the ON button and select Quit from the two choices.

Let's test your program:

You already have the function and its derivative in the proper position so press the Program key and select NEWTON and press Enter.
You will see a question mark. The program wants to know what number you want to start with. Press 1 and press Enter.

Compare your answers from section I with the answers you are getting now. They should be the same.

Exit the program. Press Program, select NEWTON and press Enter to restart the program. This time start with $x = 5$. Compare your values. How many steps does it take to get within 8 or 10 digits of accuracy on zero?

Restart the program again. Start with –5. See how many steps it takes to get the negative one root.

IV It is time to use what we have learned.

Problem Find $\sqrt[3]{3.842}$ accurate to 3 decimal places using "Newton's Method..."

Let $x = \sqrt[3]{3.842}$

Then it follows that ..... $x^3 = 3.842$ and $x^3 - 3.842 = 0$ which could be written ...

$f(x) = x^3 - 3.842$ and now we can use or program to compute the value.
Make a list of your guess and the results and the results and the results .....

Newton's Method of finding roots has been used for several purposes over the years.

One example is when you use a function which has *NO* real roots. For example $f(x) = \frac{1}{4}x^2 + 1$. This is a parabola with only a y-intercept of (0,1). You could "guess" $x = 4$ and get 1.5. Next you would get – 0.583 then 3.137, then 0.931 and so on. These values "seem" to be random and were used as a method to generate ramdon mumbers.

Another use of Newton's Method is to approximate an irrational number.
If you wanted to find an approximate value for $\sqrt{3}$ you would Let $x = \sqrt{3}$. so $x^2 = 3$
It follows that $x^2 - 3 = 0$. Now we want a zero of our equation or we could phrase it as $f(x) = x^2 - 3$.
This way we want a zero for our equation. (since $f(\sqrt{3}) = 0$).
Now you want to try a guess like $x = 2$. Find $(2, f(2))$ and $f'(2)$ etc.

Now that you are an expert with Newton's Method for finding roots, do problems 1 - 13 on page 173.

Value 3 homework assignments. (This completed worksheet is worth 2 homework assignments) and all of this is due the first day after spring break (or postmarked on that date).

Chain Rule

Open with find the $\left(\frac{1}{3x+2}\right)'$

Lecture

Pg 183

Define f' means the derivative of f WRT x if x is the variable and f is the fctn.

The 1st deriv is: $f'(x) = f' = \frac{df}{dx} = \frac{dy}{dx}$ (if fctn is called y)

$\frac{df}{dx}$ looks like a fraction and it is, sort of.

Consider $y = (3x^2+1)^2$

$y = 9x^4 + 6x^2 + 1$

$\frac{dy}{dx} = 36x^3 + 12x = 12x(3x^2+1)$

Notice how similar ↑ is to the original fctn.

$\sqrt{t}$ t $\frac{1}{2}$ no longer a variable u 5 $2t$

$12x(3x^2+1)$

$2 \cdot 3x \cdot 2(3x^2+1)$ "the inside"

"Derivat of inside" — multiply by exponent & subtract 1.

Given $y = (3x^2+1)^2$

let $u = 3x^2+1$

then $\frac{du}{dx} = 6x$

Now $y = u^2$

$\frac{dy}{du} = 2u$

Since we want $\frac{dy}{dx}$

we could say

$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du}$ ← Def of Chain Rule

$= 6x \cdot 2u$ but $u = 3x^2+1$

so $= 6x \cdot 2(3x^2+1)$

$= 12x(3x^2+1)$

Another representation is also $(f(*))' = f'(*)\,d*$

$(2x+5)^{-1}$

$f(x) = \frac{3}{2x+5}$

$= 3(2x+5)^{-1}$

$= 3(2x+5)^{-2} \cdot 2$

$\frac{-6}{(2x+5)^2}$

$(5-6x)^3$

$3(5-6x)^2(-6)$

$-18(5-6x)^2$

The Addition & Subtraction of 2 fcns and the associated derivatives are simple

For multiplication or division of two fcns we have uv and $\frac{u}{v}$ rules.

when we are faced with the composition of two fcns we use the chain rule.

using the $f(g(x))$ notation.

$$f(g(x)) = (3x^2+1)^2$$

This leads us to believe that $g(x) = 3x^2+1$ and $f(x) = x^2$.

$$g'(x) = 6x$$

$$f'(x) = 2x$$

$$(f \circ g)'(x) = f'(g(x))\, g'(x)$$

Notice this is known as the "derivative of the inside"

$$2(3x^2+1)\, 6x$$

and finally $12x(3x^2+1)$

"Take der of "the function" and multiply by der of inside"

No longer do we accept eg $\frac{1}{3x+2}$

Now you must $\frac{1}{3x+2} = (3x+2)^{-1}$

$\cdot$ der $\rightarrow -1(3x+2)^{-2}(3)$

$$\frac{(3x+2)\cdot 0 - 1(3)}{(3x+2)^2}$$

to last page

x values of

Example: find max mins ~~on I~~

$x \in [0, 4]$

$f(x) = (3x-1)^2(5-2x)^3$ ~~D_x~~

$u \quad v$

$v \quad du \quad u \quad dv$

$(5-2x)^3 \overbrace{2(3x-1)(3)}^{du} + (3x-1)^2 \overbrace{3(5-2x)^2(-2)}^{dv}$

then factor

$(5-2x)^2(3x-1)(6)\left[(5-2x) + (3x-1)(-1)\right]$

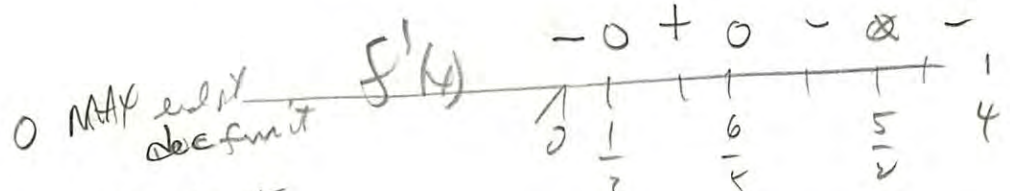
$\left[5-2x \quad -3x+1\right]$

$6(5-2x)^2(3x-1)(6-5x)$

~~Hand out~~ "Enchanted Melody"

My formula usually use to represent Chain Rule is

$(f(x))' = f'(x)\,dx$



$\frac{1}{3}$ MIN f' test

$\frac{6}{5}$ MAX f' test

4 min ENDPT done to it.

window x 0,4 1

y −125, 125, 50

unCHAINed Melody

(As Al Hibbler intended it to be...)

Find the derivatives and evaluate at the given values using the following table.
Transfer the letters to the spaces provided for a final clue for solving this puzzle.

Example 1:

$$\left(h(g(x))\right)'\Big|_{x=2} = h'(g(2)) \cdot g'(2)$$

$$h'(4) \cdot 3$$

$$3 \cdot 3 = 9$$

Example 2:

$$\left(f(g(h(x)))\right)'\Big|_{x=2} = f'(g(h(2))) \cdot g'(h(2)) \cdot h'(2)$$

$$f'(g(4)) \cdot g'(4) \cdot 2$$

$$f'(3) \cdot 2 \cdot 2$$

$$2 \cdot 2 \cdot 2 = 8$$

$x \downarrow$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
2	3	4	4	3	4	2
3	4	2	2	4	3	4
4	4	3	3	2	2	3

R) $\left(g(h(x))\right)'\Big|_{x=2} = 4$

N) $\left(g(f(x))\right)'\Big|_{x=4} = 6$

S) $\left(f(h(x))\right)'\Big|_{x=3} = 8$

F) $\left(f(g(x))\right)'\Big|_{x=2} = 9$

H) $\left(g(g(x))\right)'\Big|_{x=3} = 12$

V) $\left(h(f(x))\right)'\Big|_{x=2} = 16$

T) $\left(g(h(f(x)))\right)'\Big|_{x=3} = 18$

D) $\left(h(g(f(x)))\right)'\Big|_{x=4} = 24$

O) $\left(h(f(g(x)))\right)'\Big|_{x=2} = 27$

A) $\left(g(f(h(x)))\right)'\Big|_{x=4} =$ ~~32~~

E) $\left(f(g(f(x)))\right)'\Big|_{x=2} =$ ~~64~~

I) $\left(f(g(f(g(x))))\right)'\Big|_{x=3} =$ ~~128~~ 256

We will allow for both answers

D	E	R	I	V	A	T
24	64	4	54 / 256	16	48	18
I	V	E	O	F	T	H
~~54~~ 256	16	64	27	9	18	12
E	I	N	S	I	D	E
64	54	6	8	~~54~~ 256	24	64

H. Analysis

unCHAINed Melody

Mr. Mumaugh

(As Al Hibbler intended it to be...)

Find the derivatives and evaluate at the given values using the following table.
Transfer the letters to the spaces provided for a final clue for solving this puzzle.

Example 1:

$$\left(h\left(g(x)\right)\right)'\Big|_{x=2} = h'\left(g(2)\right)\cdot g'(2)$$

$$h'(4)\cdot 3$$

$$3\cdot 3 = 9$$

Example 2:

$$\left(f\left(g\left(h(x)\right)\right)\right)'\Big|_{x=2} = f'\left(g\left(h(2)\right)\right)\cdot g'\left(h(2)\right)\cdot h'(2)$$

$$f'\left(g(4)\right)\cdot g'(4)\cdot 2$$

$$f'(3)\cdot 2\cdot 2$$

$$2\cdot 2\cdot 2 = 8$$

$x\downarrow$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
2	3	4	4	3	4	2
3	4	2	2	4	3	4
4	4	3	3	2	2	3

R) $\left(g\left(h(x)\right)\right)'\Big|_{x=2}$

N) $\left(g\left(f(x)\right)\right)'\Big|_{x=4}$

S) $\left(f\left(h(x)\right)\right)'\Big|_{x=3}$

F) $\left(f\left(g(x)\right)\right)'\Big|_{x=2}$

H) $\left(g\left(g(x)\right)\right)'\Big|_{x=3}$

V) $\left(h\left(f(x)\right)\right)'\Big|_{x=2}$

T) $\left(g\left(h\left(f(x)\right)\right)\right)'\Big|_{x=3}$

D) $\left(h\left(g\left(f(x)\right)\right)\right)'\Big|_{x=4}$

O) $\left(h\left(f\left(g(x)\right)\right)\right)'\Big|_{x=2}$

A) $\left(g\left(f\left(h(x)\right)\right)\right)'\Big|_{x=4}$

E) $\left(f\left(g\left(f(x)\right)\right)\right)'\Big|_{x=2}$

I) $\left(f\left(g\left(f\left(g(x)\right)\right)\right)\right)'\Big|_{x=3}$

We will never forget the ...

24	64	4	256	16	48	18
256	16	64	27	9	18	12
64	256	6	8	256	24	64

① $g(h(x))'\Big|_{x=2}$

$g'(h(2))\,h'(2)$

$2 \quad 2$ (4)

4

② $(g(f(x))'\Big|_{x=4}$

$g'f(4)\,f'(4)$

$2 \cdot 3$

6

③ $f(h(x))'\Big|_{x=3}$

$f'(h(3))\,h'(3)$

$2 \cdot 4$

8

④ $f(g(x))'\Big|_{x=2}$

$f'(g(2))\,g'(2)$

$4 \quad 3$

$3 \cdot 3$

9

⑤ $(g(g(x))'\Big|_{x=3}$

$g'(g(3))\,g'(3)$

$3 \quad (2)$

$3 \cdot 4$

12

⑥ $h(f(x))'\Big|_{x=2}$

$h'(f(2))\,f'(2)$

$3 \quad 4$

$4 \cdot 4$

16

⑦ $g(h(f(x)))'\Big|_{x=3}$

$g'(h(f(3)))\,h'(f(3))\,f'(3)$

$2 \quad 4 \quad 3 \quad 4$

$3 \quad 3 \cdot 2$

18

⑧ $h(g(f(x)))'\Big|_{x=4}$

$h'(g(f(4)))\,g'(f(4))\,f'(4)$

$3 \quad 4 \quad 4 \quad 4$

$4 \quad 2 \cdot 3$

24

⑨ $h(f(g(x)))'\Big|_{x=2}$

$h'(f(g(2)))\,f'(g(2))\,g'(2)$

$4 \quad 4 \quad 3$

$3 \cdot 3$

27

⑩ $f(h(g(x)))'\Big|_{x=3}$

$f'(h(g(3)))\,h'(g(3))\,g'(3)$

$4 \quad 2$

$2 \cdot 4$

32

⑫ $f(g(f(g(x))))'\Big|_{x=3}$

$f'(g(f(g(3))))\,g'(f(g(3)))\cdot f'(g(3))\,g'(3)$

$4 \quad 4 \quad 3 \quad 2 \quad 2$

$3 \cdot 4 \quad 4 \cdot 4$

128

256

⑪ $f(g(f(x)))'\Big|_{x=2}$

$f'(g(f(2)))\,g'(f(2))\,f'(2)$

$3 \quad 3$

$2 \quad 4 \cdot 4$

$4 \cdot 64$

$f(g(f(g(x))))$

(9)

$f'(g(f(g(x)))) \cdot g'(f(g(x)) \cdot f'(g(x)) \cdot g'(x) \Big|_{x=3}$

$f(2)$, $g(3)$, $f'(2)$, 4

$g(f(g(x)))$, $g(3)$, 2

$f(g(x))$, $f(2)$, 3

$g'(3)$, 4

$f'(g(x))$, $g(3)$, $f'(2)$, 1, 4

$g'(x)$, 4

$16 \cdot 16 = 256$.

(10)

$g(f(h(x)))'$

$g'(f(h(x)) \cdot f'(h(x)) \cdot h'(x) \Big|_{x=4}$

$f(2)$, $g'(3)$, 4

$f'(2)$, 1, 4

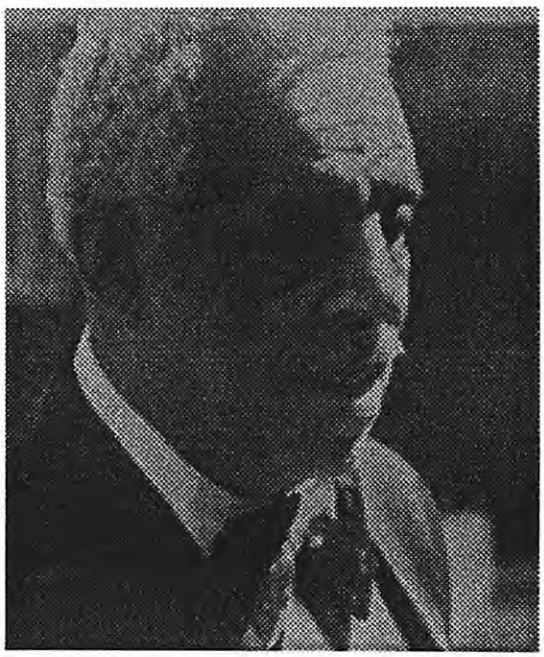
$h'(x)$, 3

48

Subject: Al Hibbler
Sent: 4/13/97 11:02 PM
Received: 4/14/97 6:17 AM
From: Mike Henry, cybthras@ix.netcom.com
To: jamesm96@concentric.net
Enclosure: hibbler.jpg

Al Hibbler was a blind, black jazz vocalist whose recordings included "September In The Rain" and "Unchained Melody." One album was "New Jersey Jazz Festival," which the songs are on.

Unchained Melody was also a prison movie, from 1955, from which the love song was based. Hibbler's was only one of several versions of the song, and it held #3 for 19 weeks.
--
Mike Henry



Jazz Profiles

Credit: Jasmine Records

playlist
coming up

about the series
program listings

Al Hibbler

Producer Joan Merrill

As a preeminent singer for the Duke Ellington band between 1943 and 1951, Al Hibbler has earned his place in the jazz arena. Blind since birth, he became the first blind artist, even before Ray Charles, to achieve real popularity as a performer. His southern accent plus his rich, deep baritone voice and emotional treatment of a song contributed to his very distinctive style. In this show we pay tribute to the irrepressible singer -- Al Hibbler.

Show Credits

The profile of Al Hibbler was written and produced by Joan Merrill. Post production engineer Stacey Abbott. Thanks to David Guerra of the University of Little Rock, Dayna Calderon of WBEZ in Chicago, the folks at KCSM in San Mateo, California and the NPR Bureaus in Los Angeles and New York.

Jazz Profiles is produced by Tim Owens. Assistant producer Madeleine Smith and Website coordinator/writer Bettina Owens. Executive Producers -- Mary Beth Kirchner and Murray Horwitz.

Albert George Hibbler was born August 16, 1915, in Tyro, Mississippi. He entered school for the first time at the Arkansas School for the Blind when he was 14 years old. There he sang in the choir as a soprano, but four years later his voice had deepened to a lush baritone. Upon leaving school, Hibbler started singing the blues in roadhouses, but he wanted to sing soft sweet ballads like crooners Russ Columbo and Bing Crosby.

Hibbler's first big break was a singing gig at a local radio station, but he was more interested in singing with big bands. Hibbler tells of arriving drunk for an audition with the Duke Ellington Orchestra. Although he didn't get that job, he eventually landed with the Jay McShann band -- one of the "swingingest" bands of the '40s, known for launching the careers of many fine musicians. "It was a gas to have Hibbler on the stand," recalls McShann. "He was outgoing and he loved people."

Hibbler stayed with McShann's band for 18 months until his second audition with Ellington. It was 8 years after his first disastrous audition, and he was ready this time. According to Hibbler, "Duke wouldn't let me go. He even wrote a song for me: 'Do Nothing 'Till You Hear From Me.'" The song was a hit

and Hibbler's dream of becoming a big band singer had come true. Hibbler was the lead singer for the Ellington Orchestra for 8 years, until a salary dispute ended the relationship. Hibbler went out on his own and signed with Decca Records. He achieved enormous success with his rendition of "Unchained Melody," which soared to the top of the charts in 1955. Author Will Friedwald talks about why he thinks that tune was so successful and why it appealed to audiences of all ages.

The tumultuous '60s ushered in the civil rights movement and Rock and Roll and the Beatles became the music of the time. Hibbler, like several other musicians, was very active in the movement, but his music career started to wane. By the '70s, recording and performing dates were infrequent, but Hibbler always remained optimistic. Today Al Hibbler still considers his career very much alive and looks forward to conquering new worlds. Maybe he'll even try acting.

Interviewees include Al Hibbler, Kay Davis, Maria Ellington, Will Friedwald, Brooks Kerr, and Jay McShann.

ABOUT THE SHOW | THIS WEEK'S PROFILE | PROGRAM LISTINGS

C 4m H Analysis

Heston's Derivative Marathon

Mr. Mumaugh

I Find the derivative of each of the following problems in set A. Locate that answer in set B . The letter with the answer goes in the blank which is marked with the problem number.

Set A

1) $(x^2-3x+8)^2$

2) $(8x-7)^{-5}$

3) $(3-2x)^{-1}$

4) $\dfrac{x}{(x^2-1)^4}$

5) $(3-2x)^2(2x-1)$

6) $\dfrac{x^2+2x}{x^2}$

7) $\left(\dfrac{3x+4}{6x-7}\right)^3$

8) $\sqrt[3]{8x^3+27}$

9) $\dfrac{5}{\sqrt[5]{x^5-32}}$

10) $\dfrac{1}{\sqrt{3x-4}}$

11) $\dfrac{x^2-4x+3}{x^{\frac{1}{2}}}$

12) $\dfrac{2x+3}{\sqrt{4x^2+9}}$

13) $\sqrt{4x^2+2x+3}$

14) $\sin(x^2+2)$

15) $\cos(4-3x)$

16) $\cos^5 3x$

17) $\sin^4 x^3$

18) $\sec(2x+1)^2$

19) $\tan(2x^2+3)$

20) $\cos(3x^2)+\cos^2 3x$

21) $\tan^3\dfrac{x}{6}$

22) $\csc^2 2x$

23) $x^2\csc^2 x^2$

24) $\tan^2 x\sec^3 x$

25) $x^2\sec^3 4x$

26) $(\sin 5x-\cos 5x)^5$

27) $\sin 4x\csc 4x$

28) $\dfrac{\cos 4x}{1-\sin 4x}$

29) $\dfrac{\sec 2x}{1+\tan 2x}$

30) $\sin\sqrt{x}+\sqrt{\sin x}$

31) $\tan\sqrt[3]{5-6x}$

32) $\cos^2\sqrt{3-8x}$

33) $\sqrt{\sin 2x-\cos 2x}$

34) $\sqrt{x^2+1}\tan\sqrt{x^2+1}$

35) $\dfrac{\cot 4x}{\sqrt{x^2+4}}$

36) $\sec\sqrt{4x+1}$

37) $\sin^2 2x\sqrt{\cos 2x}$

38) $\sin\left(\dfrac{\pi}{2}-x\right)$

39) $(2x-3)^3(3-2x)^{-1}$

40) $\sqrt{(3-x)^2-2x}$

41) $\dfrac{2(9x-2)}{135}\sqrt{(3x+1)^3}$

42) $-\dfrac{1}{3}\cos 3x+\dfrac{1}{9}\cos^3 3x$

43) $3\cos^2 4x-3\sin^2 4x$

44) $\dfrac{x}{2}-\dfrac{\sin 2x}{4}$

45) $\dfrac{1}{3}\tan 3x-x$

46) $\dfrac{1}{8}\sin 2x-\dfrac{1}{4}x\cos 2x$

47) $\dfrac{-1}{3(3x+5)}$

48) $\dfrac{1}{12}(2x^2+3)^3$

49) $\left(\dfrac{\sin x}{\sin 2x}\right)^3$

50) $\left(\dfrac{x+1}{x-1}\right)^3$

51) $\dfrac{(x+1)^2}{x-1}$

52) $\dfrac{(5-x^2)^2}{x^2-5}$

53) $\left(\dfrac{3x+2}{x+5}\right)^3$

31	42	10	25	14	35	47	7	39
9	41	22	5	21	8	34	46	1
12	49	23	18	48	30	2	50	26
6	53	37	17	43	24	45	38	11
32	13	51	27	3	36	40	15	44
19	29	52	16	33	20	28	4	!___

Part I Set B

T) $\dfrac{-2\sec^2(5-6x)^{1/3}}{(5-6x)^{2/3}}$ B) $\dfrac{1}{2}\tan^2\dfrac{x}{6}\sec^2\dfrac{x}{6}$ A) $12x^2\sin^3 x^3\cos x^3$ I) $3\sin(4-3x)$

F) $\dfrac{-4(x^2+4)\csc^2 4x - x\cot 4x}{(x^2+4)^{3/2}}$ E) $\dfrac{18-12x}{(4x^2+9)^{3/2}}$ E) $\dfrac{3}{2}x^{1/2} - \dfrac{2}{x^{1/2}} - \dfrac{3}{2x^{3/2}}$ I) $-15\sin 3x\cos^4 3x$

H) $\sin^3 3x$ E) $\dfrac{8x^2}{(8x^3+27)^{2/3}}$ N) $-48\sin 4x\cos 4x$ S) $\dfrac{1-\cos 2x}{2}$

E) $\dfrac{-3}{2(3x-4)^{3/2}}$ T) $\dfrac{x\tan\sqrt{x^2+1}}{\sqrt{x^2+1}} + x\sec^2\sqrt{x^2+1}$ D) $\sec^3 x\tan x(2\sec^2 x + 3\tan^2 x)$

D) $2x\sec^3 4x(1+6x\tan 4x)$ W) $\dfrac{1}{2}x\sin 2x$ P) $\tan^2 3x$ T) $\dfrac{2\sec 2x\,(\tan 2x - 1)}{(1+\tan 2x)^2}$

I) $2x\cos(x^2+2)$ E) $(4x-6)(x^2-3x+8)$ R) $-\sin x$ T) $2x$ A) $4x\sec^2(2x^2+3)$

F) $\dfrac{1}{(3x+5)^2}$ N) $\dfrac{3}{8}\sec^3 x\tan x$ T) $\dfrac{4\sin 2\sqrt{3-8x}}{\sqrt{3-8x}}$ T) $\dfrac{\sin 2x + \cos 2x}{\sqrt{\sin 2x - \cos 2x}}$

E) $\dfrac{-135(3x+4)^2}{(6x-7)^4}$ C) $2x\csc^2 x^2(1-2x^2\cot x^2)$ E) $\dfrac{4x+1}{\sqrt{4x^2+2x+3}}$

R) $12-8x$ O) $(8x+4)\sec(2x+1)^2\tan(2x+1)^2$ N) $\dfrac{(x-3)(x+1)}{(x-1)^2}$

E) $\dfrac{-5x^4}{(x^5-32)^{6/5}}$ N) $x(2x^2+3)^2$ D) 0 E) $\dfrac{-7x^2-1}{(x^2-1)^5}$ U) $-6x\sin 3x^2 - 3\sin 6x$

N) $x\sqrt{3x+1}$ T) $\dfrac{\sqrt{\sin x}\cos\sqrt{x} + \sqrt{x}\cos x}{2\sqrt{x\sin x}}$ E) $\dfrac{-40}{(8x-7)^6}$ E) $\dfrac{2}{(3-2x)^2}$

C) $-4\csc^2 2x\cot 2x$ N) $\dfrac{-6(x^2+2x+1)}{(x-1)^4}$ D) $25(\sin 5x - \cos 5x)^4(\sin 5x + \cos 5x)$

E) $2(3-2x)(5-6x)$ E) $\dfrac{-2}{x^2}$ R) $\dfrac{39(3x-2)^2}{(x+5)^4}$

S) $\dfrac{\sin 2x(4\cos^2 2x - \sin^2 2x)}{\sqrt{\cos 2x}}$ D) $\dfrac{4}{1-\sin 4x}$

R) $\dfrac{2\sec\sqrt{4x+1}\tan\sqrt{4x+1}}{\sqrt{4x+1}}$ S) $\dfrac{x-4}{\sqrt{x^2-8x+9}}$

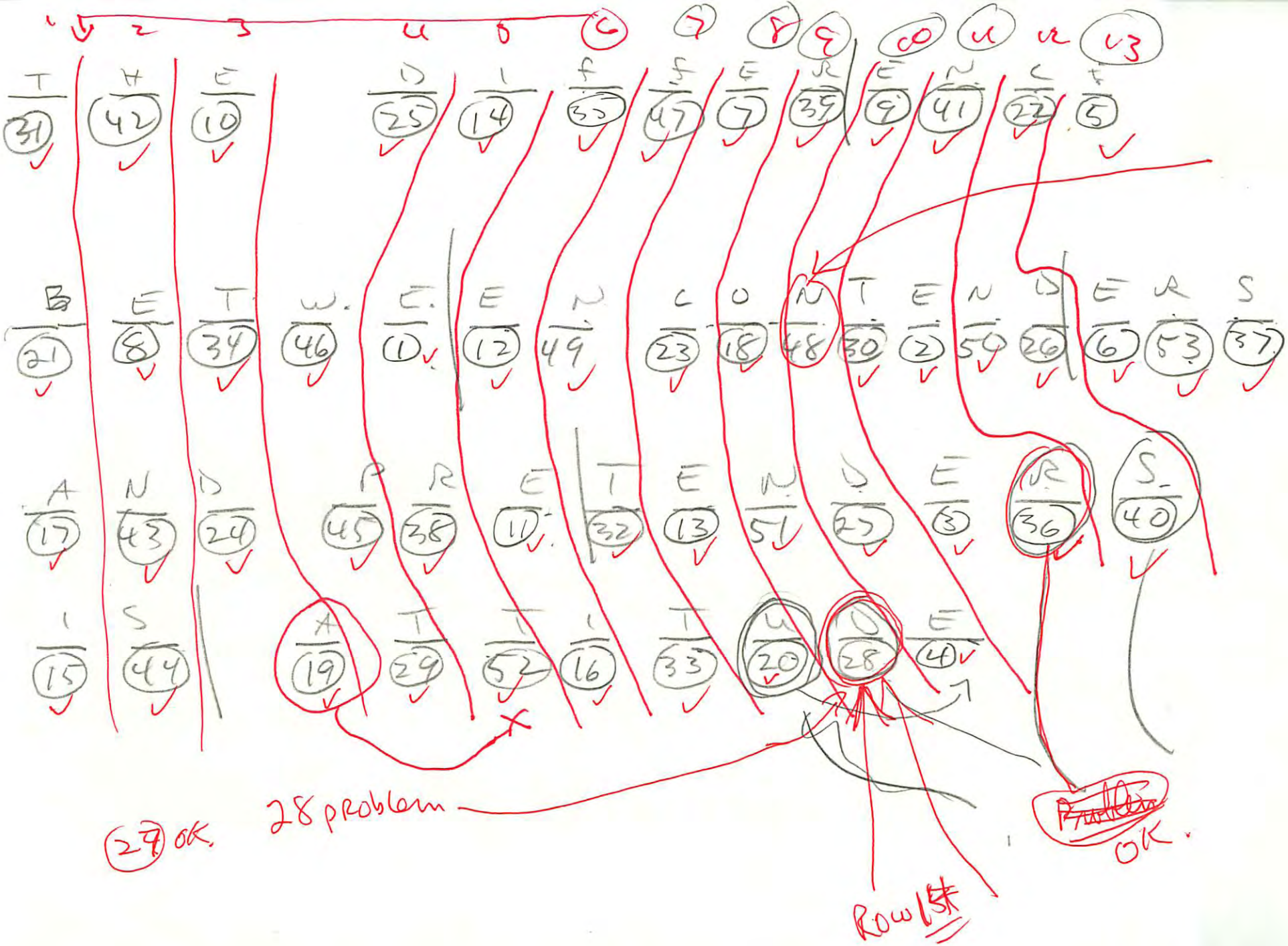
II Find the equation of the tangent line to the relation given at the given point:

a) $xy+16=0$ $(-2,8)$ b) $y^2-4x^2=5$ $(-1,3)$ c) $2x^3-x^2y+y^3-1=0$ $(1,0)$

d) $3y^4+4x-x^2\sin y-4=0$ $(1,0)$ e) $xy^2+3y=27$ $(2,3)$ f) $y=2x-\dfrac{4}{\sqrt{x}}$ $(4,6)$

III Given $x \in [0,2\pi]$. Find the x-coordinates of all points on the graph of the equation $y=3x-\cos 2x$ at which the tangent line is perpendicular to the line $2x+4y=5$.

IV If $f(x)=\sin 2x-\cos 2x$ for $0\le x\le 2\pi$, find the x-coordinates of all points on the graph of f at which the tangent line is horizontal.


THE DIFFERENCE BETWEEN AND PRETENDERS IS
28 problem
27 OK.
Row 15th
OK.

(Row, Column)

C 4m H Analysis

Heston's Derivative Marathon

Mr. Mumaugh

I Find the derivative of each of the following problems in set A. Locate that answer in set B . The letter with the answer goes in the blank which is marked with the problem number.

Set A

1) $(x^2-3x+8)^2$ (6,2)

2) $(8x-7)^{-5}$ (11,3)

3) $(3-2x)^{-1}$ (11,4)

4) $\dfrac{x}{(x^2-1)^4}$ (10,4)

5) $(3-2x)^2(2x-1)$ (13,1)

6) $\dfrac{x^2+2x}{x^2}$ (13,2)

7) $\left(\dfrac{3x+4}{6x-7}\right)^3$ (7,1)

8) $\sqrt[3]{8x^3+27}$ (3,2)

9) $\dfrac{5}{\sqrt[5]{x^5-32}}$ (10,1)

10) $\dfrac{1}{\sqrt{3x-4}}$ (4,1)

11) $\dfrac{x^2-4x+3}{x^{1/2}}$ (2,3)

12) $\dfrac{2x+3}{\sqrt{4x^2+9}}$ (2,2)

13) $\sqrt{4x^2+2x+3}$ (7,3)

14) $\sin(x^2+2)$ (6,1)

15) $\cos(4-3x)$ (1,4)

16) $\cos^5 3x$ (2,4)

17) $\sin^4 x^3$ (1,3)

18) $\sec(2x+1)^2$ (9,2)

19) $\tan(2x^2+3)$ (6,5)

20) $\cos(3x^2)+\cos^2 3x$ (10,5)

21) $\tan^3\dfrac{x}{6}$ (1,2)

22) $\csc^2 2x$ (12,1)

23) $x^2\csc^2 x^2$ (7,2)

24) $\tan^2 x\sec^3 x$ (4,3)

25) $x^2\sec^3 4x$ (5,1)

26) $(\sin 5x-\cos 5x)^5$ (12,3)

27) $\sin 4x\csc 4x$ (10,3)

28) $\dfrac{\cos 4x}{1-\sin 4x}$ (14,2)

29) $\dfrac{\sec 2x}{1+\tan 2x}$ (5,4)

30) $\sin\sqrt{x}+\sqrt{\sin x}$ (11,2)

31) $\tan\sqrt[3]{5-6x}$ (1,1)

32) $\cos^2\sqrt{3-8x}$ (7,3)

33) $\sqrt{\sin 2x-\cos 2x}$ (7,4)

34) $\sqrt{x^2+1}\tan\sqrt{x^2+1}$ (4,2)

35) $\dfrac{\cot 4x}{\sqrt{x^2+4}}$ (2,1)

36) $\sec\sqrt{4x+1}$ (15,1)

37) $\sin^2 2x\sqrt{\cos 2x}$ (14,1)

38) $\sin\left(\dfrac{\pi}{2}-x\right)$ (6,3)

39) $(2x-3)^3(3-2x)^{-1}$ (9,1)

40) $\sqrt{(3-x)^2-2x}$ (15,2)

41) $\dfrac{2(9x-2)}{135}\sqrt{(3x+1)^3}$ (11,1)

42) $-\dfrac{1}{3}\cos 3x+\dfrac{1}{9}\cos^3 3x$ (3,1)

43) $3\cos^2 4x-3\sin^2 4x$ (3,3)

44) $\dfrac{x}{2}-\dfrac{\sin 2x}{4}$ (3,4)

45) $\dfrac{1}{3}\tan 3x-x$ (5,3)

46) $\dfrac{1}{8}\sin 2x-\dfrac{1}{4}x\cos 2x$ (5,2)

47) $\dfrac{-1}{3(3x+5)}$ (7,1)

48) $\dfrac{1}{12}(2x^2+3)^3$ (10,2)

49) $\left(\dfrac{\sin x}{\sin 2x}\right)^3$ (7,2)

50) $\left(\dfrac{x+1}{x-1}\right)^3$ (12,2)

51) $\dfrac{(x+1)^2}{x-1}$ (9,3)

52) $\dfrac{(5-x^2)^2}{x^2-5}$ (6,4)

53) $\left(\dfrac{3x+2}{x+5}\right)^3$ (13,3)

31	42	10	25	14	35	47	7	39
9	41	22	5	21	8	34	46	1
12	49	23	18	48	30	2	50	26
6	53	37	17	43	24	45	38	11
32	13	51	27	3	36	40	15	44
19	29	52	16	33	20	28	4	___!___

Part I Set B

1 T) 31 $\dfrac{-2\sec^2(5-6x)^{1/3}}{(5-6x)^{1/3}}$ T) 21 $\dfrac{1}{2}\tan^2\dfrac{x}{6}\sec^2\dfrac{x}{6}$ A) 17 $12x^2\sin^3x^3\cos x^3$ I) 15 $3\sin(4-3x)$

2 F) 35 $\dfrac{-4(x^2+4)\csc^2x-x\cot 4x}{(x^2+4)^{1/2}}$ E) 12 $\dfrac{18-12x}{(4x^2+9)^{1/2}}$ E) 11 $\dfrac{3}{2}x^{1/2}-\dfrac{2}{x^{1/2}}-\dfrac{3}{2x^{3/2}}$ I) 16 $-15\sin 3x\cos^4 3x$

3 H) 42 $\sin^3 3x$ E) 8 $\dfrac{8x^2}{(8x^3+27)^{2/3}}$ N) 43 $-48\sin 4x\cos 4x$ S) 44 $\dfrac{1-\cos 2x}{2}$

4 E) 10 $\dfrac{-3}{2(3x-4)^{1/2}}$ T) 34 $\dfrac{x\tan\sqrt{x^2+1}}{\sqrt{x^2+1}}+\sec^2\sqrt{x^2+1}$ D) 24 $\sec^3x\tan x(2\sec^2x+3\tan^2x)$

5 D) 25 $2x\sec^3 4x(1+6x\tan 4x)$ W) 46 $\dfrac{1}{2}x\sin 2x$ P) 45 $\tan^2 3x$ T) 29 $\dfrac{4\sec 2x}{(1+\tan 2x)^2}$

6 I) 14 $2x\cos(x^2+2)$ E) 1 $(4x-6)(x^2-3x+8)$ R) 38 $-\sin x$ T) 52 $2x$ A) 19 $4x\sec^2(2x^2+3)$

7 F) 47 $\dfrac{1}{(3x+5)^2}$ N) 49 $\dfrac{3}{8}\sec^3x\tan x$ T) 32 $\dfrac{4\sin 2\sqrt{3-8x}}{\sqrt{3-8x}}$ T) 33 $\dfrac{\sin 2x+\cos 2x}{\sqrt{\sin 2x-\cos 2x}}$

8 E) 7 $\dfrac{-135(3x+4)^2}{(6x-7)^4}$ C) 23 $2x\csc^2x^2(1-2x^2\cot x^3)$ E) 13 $\dfrac{4x+1}{\sqrt{4x^2+2x+3}}$

9 R) 9 $12-8x$ O) 18 $(8x+4)\sec(2x+1)^2\tan(2x+1)^2$ N) 51 $\dfrac{(x-3)(x+1)}{(x-1)^2}$

10 E) 9 $\dfrac{-5x^4}{(x^5-32)^{6/5}}$ N) 48 $x(2x^2+3)^2$ D) 27 0 E) 4 $\dfrac{-7x^2-1}{(x^2-1)^5}$ U) 20 $-6x\sin 3x^2-3\sin 6x$

11 N) 41 $x\sqrt{3x+1}$ T) 30 $\dfrac{\cos\sqrt{x\sin x}+\sqrt{x}\cos x}{2\sqrt{x\sin x}}$ E) 2 $\dfrac{-40}{(8x-7)^6}$ E) 3 $\dfrac{2}{(3-2x)^2}$

12 C) 22 $-4\csc^2 2x\cot 2x$ N) 50 $\dfrac{-6(x+1)^2}{(x-1)^4}$ D) 26 $25(\sin 5x-\cos 5x)^4(\sin 5x+\cos 5x)$

13 E) 5 $2(3-2x)(5-6x)$ E) 6 $\dfrac{-2}{x^2}$ R) 53 $\dfrac{39(3x-2)^2}{(x+5)^4}$

14 S) 37 $\dfrac{\sin 2x(4\cos^2 2x-\sin^2 2x)}{\sqrt{\cos 2x}}$ D) 28 $\dfrac{4}{1-\sin 4x}$

15 R) 36 $\dfrac{2\sec\sqrt{4x+1}\tan\sqrt{4x+1}}{\sqrt{4x+1}}$ S) 40 $\dfrac{x-4}{\sqrt{x^2-8x+9}}$

II

Find the equation of the tangent line to the relation given at the given point:

a) $xy+16=0$ $(-2,8)$ b) $y^2-4x^2=5$ $(-1,3)$ c) $2x^3-x^2y+y^3-1=0$

d) $3y^4+4x-x^2\sin y-4=0$ $(1,0)$ e) $xy^2+3y=27$ $(2,3)$ f) $y=2x-\dfrac{4}{\sqrt{x}}$ $(4,6)$

III

Find the x-coordinates of all points on the graph of the equation $y=3x-\cos 2x$ at which the tangent line is perpendicular to the line $2x+4y=5$.

IV

If $f(x)=\sin 2x-\cos 2x$ for $0\le x\le 2\pi$, find the x-coordinates of all points on the graph of f at which the tangent line is horizontal.

(1) $(x^2-3x+8)^2$

E

$2(x^2-3x+8)(2x-3)$

(2) $(8x-7)^{-5}$ E

$\frac{-40}{(8x-7)^6}$

(3) $(3-2x)^{-1}$ E

$\frac{2}{(3-2x)^2}$

(4) $\frac{x}{(x^2-1)^4}$ E

$\frac{(x^2-1)^4 - x(4(x^2-1)^3 2x)}{(x^2-1)^8}$

$\frac{x^2-1-8x^2}{(x^2-1)^5}$

$\frac{-1-7x^2}{(x^2-1)^5}$

(5) $(3-2x)^2(2x-1)$ E

$(2x-1)2(3-2x)(-2) + (3-2x)^2 2$

$(3-2x)[-4(2x-1) + 2(3-2x)]$

$(3-2x)(-8x+4+6-4x)$

$(3-2x)(10-12x)$

(6) $\frac{x^2+2x}{x^2}$ E

$\frac{(x^2)(2x+2) - (x^2+2x)2x}{x^4}$

$\frac{2x^3+2x^2-2x^2-4x}{x^4}$

$\frac{-2x}{x^{13}}$

$\frac{-2}{x^2}$

$\frac{x+2}{x}$

$\frac{x-(x+2)}{x^2}$

$\frac{-2}{x^2}$

(7) $\left(\frac{3x+4}{6x-7}\right)^3$ E

$3\left(\frac{3x+4}{6x-7}\right)^2\left[\frac{(6x-7)3-(3x+4)6}{(6x-7)^2}\right]$

$\frac{3}{(6x-7)^3}(18x-21-18x-24)$

$\frac{-135(3x+4)^2}{(6x-7)^4}$

(8) $(8x^3+27)^{\frac{1}{3}}$ ✓ E

$\frac{1}{3}\cdot 24x^2(8x^3+27)^{-\frac{2}{3}}$

$\frac{8x^2}{(8x^3+27)^{2/3}}$

(9) $5(x^5-32)^{-\frac{1}{5}}$ ✓ E

$\frac{-5x^4}{(x^5-32)^{6/5}}$

(10) $(3x-4)^{-\frac{1}{2}}$ ✓ E

$\frac{-3}{2(3x-4)^{3/2}}$

(11) $\frac{x^2-4x+3}{x^{\frac{1}{2}}}$ ✓ E

$x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$

$\frac{3x^{\frac{1}{2}}}{2} + \frac{2}{x^{\frac{1}{2}}} - \frac{3}{2x^{\frac{3}{2}}}$

(12) $\frac{2x+3}{\sqrt{4x^2+9}}$ ✓ E

$\frac{2\sqrt{4x^2+9} - \frac{(2x+3)8x}{2\sqrt{4x^2+9}}}{4x^2+9}$

$\frac{2(4x^2+9) - 8x^2 - 12x}{(4x^2+9)^{3/2}}$

$\frac{18-12x}{(4x^2+9)^{3/2}} = \frac{6(3-2x)}{(4x^2+9)^{3/2}}$

(13) E ✓ $(4x^2+2x+3)^{\frac{1}{2}}$

$\frac{8x+2}{2\sqrt{4x^2+2x+3}}$

$\frac{4x+1}{\sqrt{4x^2+2x+3}}$

(14) I ✓ $\sin(x^2+2)$

$2x\cos(x^2+2)$

(15) I ✓ $\cos(4-3x)$

$3\sin(4-3x)$

(16) ✓ I $\cos^5 3x$

$-5\cos^4 3x \sin 3x \cdot 3$

$-15\sin 3x \cos^4 3x$

(17) ✓ A $\sin^4 x^3$

$4\sin^3 x^3 \cos x^3 \cdot 3x^2$

$12x^2 \sin^3 x^3 \cos x^3$

(18) O ✓ $\sec(2x+1)^2$

$4(2x+1)\sec(2x+1)^2 \tan(2x+1)^2$

(19) ✓ A $\tan(2x^2+3)$

$4x\sec^2(2x^2+3)$

(20) ✓ U $\cos 3x^2 + \cos^2 3x$

$-6x\sin 3x^2 + 6\cos 3x \sin 3x$

$-6x\sin 3x^2 - 3\sin 6x$

(21) ✓ B $\tan^3 \frac{x}{6}$

$\frac{3}{6}\tan^2\frac{x}{6}\sec^2\frac{x}{6}$

$\frac{1}{2}\tan^2\frac{x}{6}\sec^2\frac{x}{6}$

(22) ✓ C $\csc^2 2x$

$-2\cdot 2\csc^2 2x \cot 2x$

$-4\csc^2 2x \cot 2x$

(23) ✓ $x^2 \csc^2 x^2$

$2x\csc^2 x^2 - x^2 \cdot 2\csc x^2 \cot x^2 (2x)$

$2x\csc^2 x^2 (1 - 2x^2 \cot x^2)$

(24) ✓ D $\tan^2 x \sec^3 x$

$\sec^3 x\, 2\tan x \sec^2 x + \tan^2 x\, 3\sec^2 x \sec x \tan x$

$2\tan x \sec^5 x + 3\tan^3 x \sec^3 x$

$\tan x \sec^3 x (2\sec^2 x + 3\tan^2 x)$

(25) ✓ D $x^2 \sec^3 4x$

$2x\sec^3 4x + 12x^2 \sec^2 4x \sec 4x \tan 4x$

$2x\sec^3 4x (1 + 6x\tan 4x)$

(26) ✓ D $(\sin 5x - \cos 5x)^5$

$5(\sin 5x - \cos 5x)^4 (5\cos 5x + 5\sin 5x)$

(27) D $\sin 4x \csc 4x$

1

derivative of constant

0

(28) $\frac{\cos 4x}{1-\sin 4x}$ u ✓ v

$\frac{(1-\sin 4x)(-4\sin 4x) - \cos 4x(-4\cos 4x)}{(1-\sin 4x)^2}$

$\frac{-4\sin 4x + 4\sin^2 4x + 4\cos^2 4x}{(1-\sin 4x)^2}$ $\quad 4 - 4\sin^2 4x$

$\frac{4(1-\sin 4x)}{(1-\sin 4x)^2} = \frac{4}{1-\sin 4x}$

(29) T $\frac{\sec 2x}{1+\tan 2x}$ u / v

$$\frac{(1+\tan 2x)\,2\sec 2x\tan 2x - 2\sec^3 2x}{(1+\tan 2x)^2}$$

$$\frac{2\sec 2x\tan 2x + 2\sec 2x\tan^2 2x - 2\sec^3 2x}{(1+\tan 2x)^2} \qquad (\sec^2 2x - 1)$$

$$\frac{2\sec 2x\tan 2x + 2\sec^3 2x - 2\sec 2x - 2\sec^3 2x}{(1+\tan 2x)^2}$$

$$\frac{2\sec 2x(\tan 2x + 1)}{(1+\tan 2x)^2}$$

(30) T $\sin\sqrt{x} + \sqrt{\sin x}$

$$\frac{\cos\sqrt{x}}{2\sqrt{x}} + \frac{\cos x}{2\sqrt{\sin x}}$$

$$\frac{\cos\sqrt{x}\sqrt{\sin x} + \sqrt{x}\cos x}{2\sqrt{x\sin x}}$$

(31) T $\tan(5-6x)^{\frac{1}{3}}$

$$\frac{-2\sec^2(5-6x)^{\frac{1}{3}}}{3(5-6x)^{2/3}}$$

(32) T $\cos^2\sqrt{3-8x}$

$$(+8)\,2\cos\sqrt{3-8x}\,\sin\sqrt{3-8x}$$

$$\frac{2\sqrt{3-8x}}{}$$

$$\frac{4\sin 2\sqrt{3-8x}}{\sqrt{3-8x}}$$

(33) T $\sqrt{\sin 2x - \cos 2x}$

$$\frac{2(\cos 2x + \sin 2x)}{2\sqrt{\sin 2x - \cos 2x}}$$

$$\frac{\cos 2x + \sin 2x}{\sqrt{\sin 2x - \cos 2x}}$$

(34) T $\sqrt{x^2+1}$ u, $\tan\sqrt{x^2+1}$ v

$$\frac{2x\tan\sqrt{x^2+1}}{2\sqrt{x^2+1}} + \frac{\sqrt{x^2+1}\sec^2\sqrt{x^2+1}\cdot 2x}{2\sqrt{x^2+1}}$$

$$\frac{x\tan\sqrt{x^2+1} + \sqrt{x^2+1}\sec^2\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

(35) F $\frac{\cot 4x}{\sqrt{x^2+4}}$ u / v

$$\frac{\sqrt{x^2+4}\,(-\csc^2 4x)(4) - \cot 4x\cdot x}{(\sqrt{x^2+4})^2}$$

$$\frac{-(x^2+4)\csc^2 4x - x\cot 4x}{(x^2+4)^{3/2}}$$

(36) R $\sec\sqrt{4x+1}$

$$\frac{2\sec\sqrt{4x+1}\tan\sqrt{4x+1}}{\sqrt{4x+1}}$$

(37) S $\sin^2 2x$ u, $\sqrt{\cos 2x}$ v

$$\sqrt{\cos 2x}\,2\sin 2x\cos 2x\cdot 2 + \frac{\sin^2 2x\,(-\sin 2x)\,2}{2\sqrt{\cos 2x}}$$

$$1-\sin^2 2x$$

$$\frac{4\sin 2x\cos^2 2x + \sin^3 2x}{\sqrt{\cos 2x}}$$

$$\frac{4\sin 2x - 5\sin^3 2x}{\sqrt{\cos 2x}}$$

(38) R
$\sin\left(\frac{\pi}{2}-x\right)$
$-\cos\left(\frac{\pi}{2}-x\right)$
$-\left[\cos\frac{\pi}{2}\cos x+\sin\frac{\pi}{2}\sin x\right]$
$-\sin x$

(39) R
$(2x-3)^3(3-2x)^{-1}$
$\frac{(2x-3)^3}{-(2x-3)}=-(2x-3)^2$
$-2(2x-3)2$
$-8x+12$

(40) S
$\sqrt{(3-x)^2-2x}$
$\sqrt{9-6x+x^2-2x}$
$\sqrt{x^2-8x+9}$
$\frac{2x-8}{2\sqrt{x^2-8x+9}}$
$\frac{x-4}{\sqrt{x^2-8x+9}}$

(41) N
$\frac{2(9x-2)}{135}\sqrt{(3x+1)^3}$
$(3x+1)^{3/2}\frac{18}{135}+\frac{2(9x-2)}{135}\frac{3}{2}(3x+1)^{\frac{1}{2}}\cdot 3$
$\frac{(3x+1)^{\frac{1}{2}}}{135}\left[(3x-1)18+9(9x+2)\right]$
$\left[54x-18+81x+18\right]$
$\frac{135x(3x+1)^{\frac{1}{2}}}{135}$
$x\sqrt{3x+1}$

(42) H
$\frac{1}{3}\cos 3x+\frac{1}{9}\cos^3 3x$
$1-\sin^2 3x$
$\sin 3x-\cos^2 3x\sin 3x$
$\sin 3x-\sin 3x+\sin^3 3x$
$\sin^3 3x$

(43) N
$3\cos^2 4x-3\sin^2 4x$
$3\cos 8x$
$-24\sin 8x$
$-48\sin 4x\cos 4x$

(44) S
$\frac{x}{2}-\frac{\sin 2x}{4}$
$\frac{1}{2}-\frac{1}{2}\cos 2x$
$\frac{1-\cos 2x}{2}$

(45) P
$\frac{1}{3}\tan 3x-x$
$\sec^2 3x-1$
$\tan^2 3x$

(46) W
$\frac{1}{8}\sin 2x-\frac{1}{4}x\cos 2x$
$\frac{1}{4}\cos 2x-\frac{1}{4}\left[\cos 2x-2x\sin 2x\right]$
$-\frac{1}{4}\cos 2x+\frac{x}{2}\sin 2x$
$\frac{x}{2}\sin 2x$

(47) F
$\frac{-1}{3(3x+5)}$
$-\frac{1}{3}(3x+5)^{-1}$
$\frac{1}{(3x+5)^2}$

(48) N
$\frac{1}{12}(2x^2+3)$
$\frac{1}{4}(2x^2+3)^2 4x$
$x(2x^2+3)^2$

(49) N
$\left(\frac{\sin x}{\sin 2x}\right)^3$
$\left(\frac{\sin x}{2\sin x\cos x}\right)^3=\left(\frac{1}{8}\cos^{-3}x\right.$
$-\frac{3}{8}\cos^{-4}x(-\sin x$
$\frac{3\sin x}{8\cos^4 x}$

(50) N

$\left(\frac{x+1}{x-1}\right)^3$ u v

$y-1 \quad -x-1$

$3\left(\frac{x+1}{x-1}\right)^2\left[\frac{(x-1)-(x+1)}{(x-1)^2}\right]$

$\frac{-6(x+1)^2}{(x-1)^4} \quad \frac{-6x^2-12x-6}{(x-1)^4}$

(51) N

$\frac{(x+1)^2}{x-1}$ u v

$\frac{(x-1)2(x+1)-(x+1)^2}{(x-1)^2}$

$\frac{2x^2-2-x^2-2x-1}{(x-1)^2}$

$\frac{x^2-2x-3}{(x-1)^2} = \frac{(x-3)(x+1)}{(x-1)^2}$

(52) T

$\frac{(5-x^2)^2}{x^3-5}$

$\frac{-(5-x^2)^2}{5-x^2}$

$-(5-x^2)$

$-5+x^2$

$2x$

(53)

$\left(\frac{3x+2}{x+5}\right)^3$ u v

$3\left(\frac{3x+2}{x+5}\right)^2\left[\frac{3(x+5)-(3x+2)}{(x+5)^2}\right]$

$\frac{3(3x+2)^2[3x+15-3x-2]}{(x+5)^4}$

$\frac{39(3x+2)^2}{(x+5)^4}$

Industrial Strength Derivatives

Mr. Mumaugh

A **For each of the following, find the first derivative two times. First use the product rule. Next, clear parentheses in the original expression and simplify. Then, take the derivative in the usual manner. Reconcile any differences in final answers.**

1) $8x(3x+2)$ 2) $2x^3\left(x-x^3\right)$ 3) $\left(3x^2+2x\right)(5x-2)$ 4) $3x^3(4x+8)$

5) $3x^2\left(x-\sqrt{x}\right)$ 6) $x^2\left(x^2+x^2\right)$ 7) $(3x+3)(3x-3)$ 8) $\left(2-3x^2\right)\left(x^2+2\sqrt{x}\right)$

9) $(2x+1)(1-2x)$ 10) $(2x+3)^3$ 11) $(2x+3)^4$ 12) $(2-3x)^3$

B **Use the quotient rule to find the derivative:**

1) $\dfrac{(2x-1)}{3x-2}$ 2) $\dfrac{(2x-1)^2}{3x^2-2}$ 3) $\dfrac{(2x-1)}{3x^2}$ 4) $\dfrac{2x-3}{x}$

5) $\dfrac{ax+d}{c-dx}$ 6) $\dfrac{3x-5}{5-3x}$ 7) $\dfrac{3x+2}{8-12x}$ 8) $\dfrac{5x^2+2}{2x^5-5}$

C **Find the derivative:**

1) $\dfrac{3x(2x-1)}{5x+2}$ 2) $\dfrac{(3x+2)(2x-3)}{(5x+1)}$ 3) $\dfrac{(3x+2)(2x-3)}{(3-2x)(5x+1)}$ 4) $\dfrac{(x+2)(3x-4)}{(x-2)(5x+1)}$

II **A** **Find the derivative twice. First clear parentheses and use the "quick derivative." Second, use the chain rule. Clear parentheses to verify that you got the same derivative.:**

1) $(3x-2)^2$ 2) $\left(3\sqrt{x}-2\right)^2$ 3) $\left(2x^2+4x+1\right)^2$ 4) $(2x+1)^4$

B **Use the chain rule to find the derivative. Note that some problems may require a combination of the product rule, quotient rule as well as the chain rule. Be sure that your work makes it perfectly clear what method you used! Show the "u"s and the "v"s.**

1) $(3x-1)^2(5-2x)^3$ 2) $\dfrac{\left(1-3x^2\right)^3}{(4x)^2}$ 3) $\dfrac{\left(x^2+1\right)^3}{3x-1}$ 4) $\dfrac{1}{(2x+3)^2}$

5) $\dfrac{1}{(2x+3)}$ 6) $x\left(\dfrac{x^2}{x+1}\right)$ 7) $\left(\dfrac{1-2x}{2+x}\right)^2$ 8) $\dfrac{(1-2x)^2}{(2+x)^2}$

9) $\sqrt{4x^2+2x+1}$ 10) $\sqrt{5-3x^2}$ 11) $\dfrac{1}{\sqrt{5-3x^2}}$ 12) $\dfrac{3x}{\sqrt{1-x^2}}$

13) $\dfrac{(x+1)^2(x-1)^2}{\left(x^2-1\right)^2}$ 14) $x(x-1)\left(x^2-x+1\right)$ 15) $\dfrac{x^2+1}{x^2}$ 16) $1+\dfrac{1}{x^2}$

17) $\sqrt{\dfrac{2x-5}{3x-8}}$ 18) $(2+3)^{17}$ 19) $\dfrac{x^2-1}{x-1}$ 20) $\dfrac{x^3-1}{x-1}$

21) $\left((2x-1)^2+2x\right)^2$ 22) $\left(4x^2-2x+1\right)^2$ 23) $16x^4-16x^3+9x^2-8x+1$

more Industrial Strength Derivatives

III Find $\frac{dy}{dx}$

a) $x^2 + y^2 = 16$

b) $x^2+y^2-5x-8y-14=0$

c) $xy^2 = x+y$

d) $x^{2/3} + y^{2/3} = 8^{2/3}$

e) $x^2 - y^2 = 1$

f) $x^2 - 2xy + 4y^2 - 4x = 0$

g) $y^2 = \frac{x^2}{x^2+4}$

h) $5x^2 + 4xy + 3y^2 + y = 0$

i) $(x+y)^2+(x-y)^2=16+2y$

j) $4xy+x^2+2y^2=25$

k) $x^2 = \frac{x-y^2}{x+y}$

l) $\sqrt{x+y}+\sqrt{x+2y}=10$

IV There are six standard trig functions. What are their derivatives? (hint: $\sec x = \frac{1}{\cos x} = (\cos x)^{-1}$)

V Find $f'(x)$ two times. First take the derivative of the given as it stands. Second, change the given using a trig identity and then take the derivative. Evaluate both of your answers at $x = \frac{\pi}{4}$.

a) $f(x) = 2 \sin 2x \cos 2x$

b) $f(x) = \frac{1}{2} \sin 4x$

c) $f(x) = \frac{\tan^4 x}{\sec^2 x}$

d) $f(x) = \sin^2 2x + \cos^2 2x$

e) $f(x) = \frac{\sin 2x}{\cos x}$

f) $f(x) = (\sin^2 x + \cos^2 x)$

g) $f(x) = 1 - \sin^2 x$

h) $f(x) = \sin 2x$

i) $f(x) = 2 \sin x \cos x$

j) $f(x) = \frac{1}{2}(1 + \cos 2x)$

k) $f(x) = \frac{1}{2}(1 - \cos 2x)$

l) $f(x) = (\sin x + \cos x)^2$

m) $f(x) = \frac{\cos^2 x}{1 - \sin x}$

n) $f(x) = \sin x \tan x + \cos x$

o) $f(x) = \frac{1 + 2 \sin x \cos x}{\sin x + \cos x}$

p) $f(x) = \frac{1 - \cos 2x}{\sin 2x}$

q) $f(x) = \sec^2 x$

r) $f(x) = \frac{2 \tan x}{1 + \tan^2 x}$

VI Find $f'(x)$. Do not simplify. Be sure to do all the pieces in the proper order.

a) $f(x) = \left(\frac{\sqrt{3x^2 - x}}{5x^3 + 1}\right)^{\frac{2}{3}}$

b) $f(x) = \frac{(3x^3 + 2)(2x - 3)^2}{\sqrt{5x^3 + 1}}$

c) $f(x) = \frac{5(\sin^2 x - \cos x)^3}{6\sqrt{\sin x \cos x}}$

VII Prepare proper numberlines for each of the following. Sketch a graph of f. Identify extrema and PI. Justify as usual. The stated intervals are "windows of view" and not necessarily the valid domains.of the functions.

a) $f(x) = \sin x + x \; ; \; -2\pi \le x \le 2\pi$

b) $f(x) = \sin x + \cos x \; ; \; -2\pi \le x \le 2\pi$

c) $f(x) = \frac{\cos x}{1 + \sin x} \; ; \; -2\pi \le x \le 2\pi$

d) $f(x) = \sin\left(\frac{x^2}{1 + x^2}\right) \; ; \; -2\pi \le x \le 2\pi$

e) $f(x) = \sqrt{x - x^2} \; ; \; -2 \le x \le 2$

f) $f(x) = x^6 - x^4 \; ; \; -2 \le x \le 2$

Implicit

$x^2 + y^2 = 25$

$2x \frac{dx}{dx} + 2y \frac{dy}{dx} = 0$

$2x + 2y \frac{dy}{dx} = 0$

$x + y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{y}$ $\Big|_{3,4}$

$= -\frac{3}{4}$

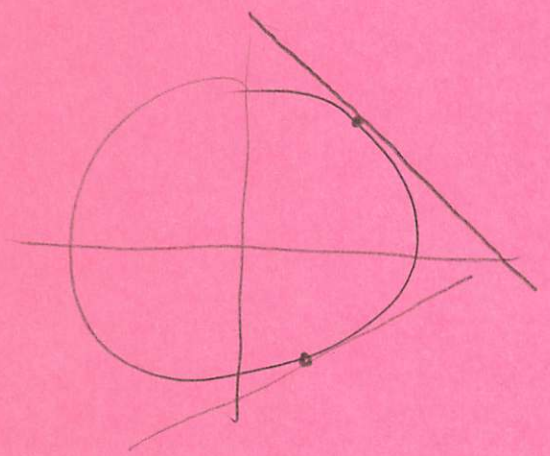
Explicit

$y = \sqrt{25 - x^2}$

$\frac{dy}{dx} = \frac{-2x}{2\sqrt{25-x^2}}$

$= \frac{-x}{\sqrt{25-x^2}}$

(3, 4)



$\frac{-3}{\sqrt{25-16}}$

$-\frac{3}{4} = m$

what about at (3, -4)?

In explicit we must use $-\sqrt{25-x^2}$ because the y value is negative

in implicit, we just substitute our values.

$(1,-1)$

$$(3x^2-y)^2 + 5x^2y^3 - 3y^4 = 8$$

$$2(3x^2-y)\left(6x - \frac{dy}{dx}\right) + 10xy^3 + 15x^2y^2\frac{dy}{dx} - 12y^3\frac{dy}{dx} = 0$$

$$2(3+1)\left(6 - \frac{dy}{dx}\right) - 10 + 15\frac{dy}{dx} + 12\frac{dy}{dx} = 0$$

4

8

$$48 - 8\frac{dy}{dx} - 10 + 27\frac{dy}{dx} = 0$$

$$19\frac{dy}{dx} = -38$$

$$\frac{dy}{dx} = -2$$

$$y+1 = -2(x-1)$$

$$= -2x+2$$

$$y = -2x+1$$

Given: $x\sqrt{y+1} = xy + 1$

find equations of tangent line to the curve at each axis intercept and when $x = -1$

$$x\sqrt{y+1} = xy + 1$$

(2)

Then. $\sqrt{y+1} + \frac{x}{2\sqrt{y+1}}\frac{dy}{dx} = y + x\frac{dy}{dx}$

1ST do this

when $x = -1$

$-\sqrt{y+1} = -y+1$

$y+1 = y^2 - 2y + 1$

$y^2 - 3y = 0$

$y(y-3) = 0$

$y = 0 \quad y = 3$

↑ Reject Extraneous Root.

So PT is $(-1, 3)$

$$2 - \frac{1}{2\cdot 2}\frac{dy}{dx} = 3 - \frac{dy}{dx}$$

$$\frac{3}{4}\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{4}{3} = m$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{4}{3}(x+1)$$

$$y = \frac{4}{3}x + \frac{4}{3} + \frac{9}{3}$$

$$y = \frac{4}{3}x + \frac{13}{3}.$$

---

let $x=0$ Doesn't cross y axis.

let $y=0$ $x = 1$.

$(1, 0)$ $\quad \frac{-2}{1-2} = +2 = m$

This example doesn't highlight implicit well enough.

find equation line to

$$\overset{u}{x}\overset{v}{\sqrt{y+1}} = \overset{u}{x}\overset{v}{y+1} \quad \text{when } x=-1$$

$$\sqrt{y+1} + \frac{x}{2\sqrt{y+1}}\frac{dy}{dx} = y + x\frac{dy}{dx}$$

$$\left(\frac{x}{2\sqrt{y+1}} - x\right)\frac{dy}{dx} = y - \sqrt{y+1}$$

Not Necessary

$$\left(\frac{x - 2x\sqrt{y+1}}{2\sqrt{y+1}}\right)\frac{dy}{dx} =$$

$$\frac{dy}{dx} = \frac{\left(y - \sqrt{y+1}\right)2\sqrt{y+1}}{\left(x - 2x\sqrt{y+1}\right)}$$

$$\frac{dy}{dx} = \frac{2y\sqrt{y+1} - 2y - 2}{x - 2x\sqrt{y+1}}$$

$$2 - \frac{1}{4}\frac{dy}{dx} = 3 - \frac{dy}{dx}$$

$$\frac{3}{4}\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{4}{3}$$

when $x=-1$

$$-\sqrt{y+1} = -y+1$$

$$y+1 = y^2 - 2y + 1$$

$$y^2 - 3y = 0$$

$$y(y-3) = 0$$

$$y \in \{0, 3\}$$

But $y=0$ is extraneous root

line #1

$(-1, 0)$

$$\frac{dy}{dx} = \frac{2\cdot 0 - 0 - 1}{-1+2}$$

$$= \frac{-1}{1}$$

$$= -1$$

$$y = -1(x+1)$$

$$y = -x - 1$$

line #2

$(-1, 3)$

$$\frac{dy}{dx} = \frac{6\cdot 2 - 6 - 2}{-1 + 2\cdot 2}$$

$$\frac{12-8}{3}$$

$$\frac{dy}{dx} = \frac{4}{3}$$

$$y - 3 = \frac{4}{3}(x+1)$$

$$= \frac{4}{3}x + \frac{4}{3} - \frac{9}{3}$$

$$y = \frac{4}{3}x - \frac{1}{3}$$

$\frac{x^2+1}{x}$

#23 $f(x) = x + \frac{1}{x}$

$f(x) = x + x^{-1}$

$f'(x) = 1 - x^{-2} \overset{\text{set}}{=} 0$

$\frac{1}{x^2} = 1$

$x^2 = 1$

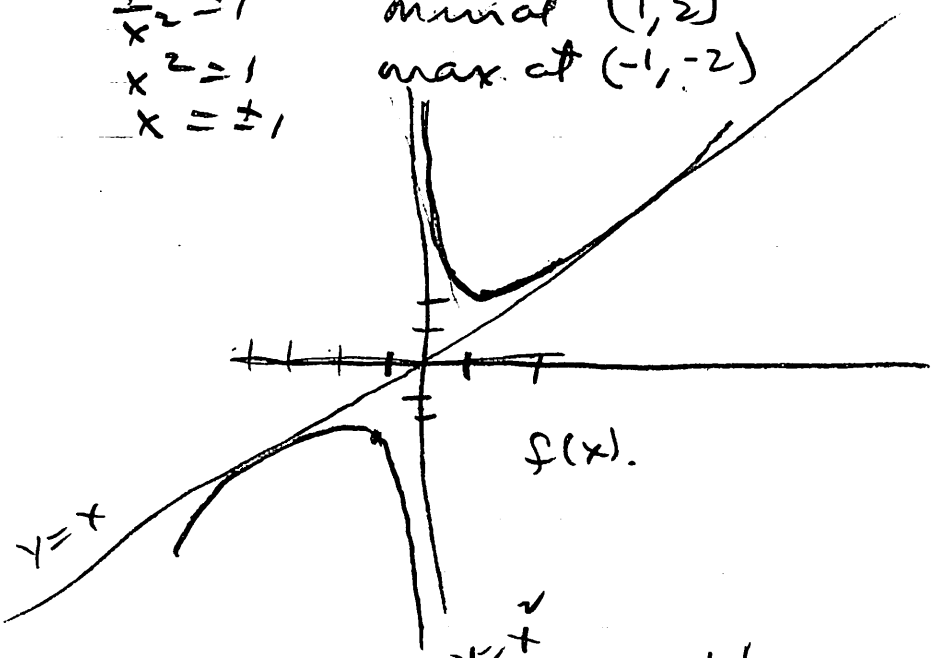
$x = \pm 1$

$f''(x) = \frac{1}{x^3}$

$f''(x) > 0 \quad \forall x > 0$

min at $(1, 2)$

max at $(-1, -2)$



#24

$f(x) = x^2 + \frac{1}{x^2} = x^2 + x^{-2}$

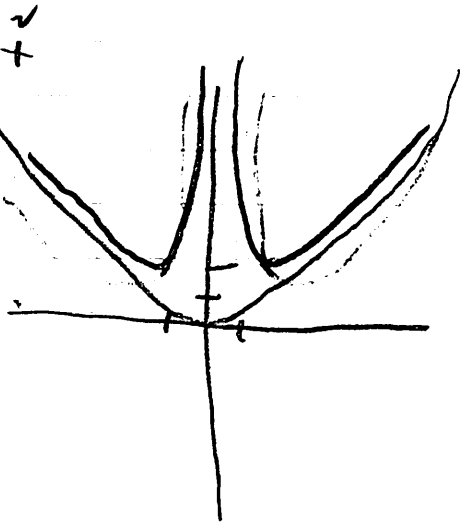
$f'(x) = 2x - 2x^{-3} \overset{\text{set}}{=} 0 \qquad 2x = \frac{2}{x^3}$

$f''(x) = 2 + 6x^{-4} \qquad x^4 = 1$

$f''(x) > 0$ ∀ x $\qquad x = \pm 1$

min at $x = \pm 1 \qquad f(\pm 1) = 1 + 1 = 2$

thus min is 2.



#25

$pq = c \qquad q = \frac{c}{p}$ c is a constant. $p, q > 0$

$f(p) = p^2 + \frac{c^2}{p^2} \qquad f'(p) = 2p - \frac{2c^2}{p^3} \overset{\text{set}}{=} 0$

$2p^4 - 2c^2 = 0$

$p^4 = c^2$

$p = \sqrt{c}$

$f(\sqrt{c}) = c + \frac{c^2}{c} = 2c$

Newton's method

§169 - 173

$g(x) = x^3 - 3x - 1$

$g(1) = 1 - 3 - 1 = -3$

$g(2) = 8 - 6 - 1 = 1$

$\therefore$ root between $x=1, x=2$

Let x_1 = 1ST guess.

$(x_1, g(x_1))$ is the pt on the curve

that point has a slope $m = g'(x_1)$

So the equation of the line:

$$y - y_1 = m(x - x_1)$$

becomes

$$y - g(x_1) = g'(x_1)(x - x_1)$$

but we want the x-intercept of this tangent line to be our 2nd guess so $y = 0$

to get

$$0 - g(x_1) = g'(x_1)(x - x_1)$$

and we solve for x

$$-g(x_1) = g'(x_1)x - g'(x_1)x_1$$

$$x_1 g'(x_1) - g(x_1) = g'(x_1)x$$

$$\frac{x_1 g'(x_1) - g(x_1)}{g'(x_1)} = x$$

$$x_i - \frac{g(x_i)}{g'(x_i)} = x_{i+1}$$

$$x_1 - \frac{g(x_1)}{g'(x_1)} = x$$

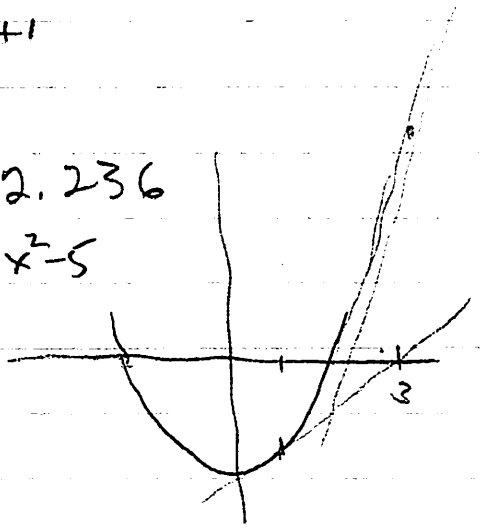
↑ this x is the NEXT choice, ie x_2

Thus — to generalize

$$x_N - \frac{g(x_N)}{g'(x_N)} = x_{N+1}$$

Pblm 2 Value of $\sqrt{5} \approx 2.236$

$x^2 - 5 = 0 \Rightarrow f(x) = x^2 - 5$



$f' = 2x = 0$

Critical value at $x = 0$

$\therefore$ choose $x_N > 0$.

1st guess $x = 1$

$$1 - \frac{-4}{2}$$

2nd guess $1 + 2 = 3.$

$$3 - \frac{4}{6}$$

2nd guess $3 - \frac{2}{3} = 2.3\overline{3}$

$$2.33 - \frac{.44}{4.66}$$

4th guess $2.33 - .095 = 2.238$

(7) $f(x) = x^3 - 3x + 1 \quad (0, 1)$

$f'(x) = 3x^2 - 3$

1st $x = 0 \quad x = 0 - \frac{1}{-3} = \frac{1}{3}$

2nd $x = \frac{1}{3} \quad x = \frac{1}{3} - \frac{.037037}{2.66}$

$-.0138$

3rd $.3194$

4th $.3194 \longrightarrow .3470124$

$.34727$

(11) $y = x^3 \qquad 2x + y = 2$

$x^3 \overset{\text{set}}{=} y = -2x + 2$

$f(x) = x^3 + 2x - 2$ Now get x value

(12) $x_{N+1} = \frac{1}{2}\left(x_N + \frac{a}{x_N}\right) \quad f = \sqrt{a}$

$f' = \frac{1}{2\sqrt{a}}$

$x_{N+1} = \sqrt{a} - \frac{\frac{1}{2\sqrt{a}}}{\sqrt{a}}$

$= \sqrt{a} - \frac{1}{2\sqrt{a}}$

$= \frac{2a - 1}{2\sqrt{a}}$

$= \frac{1}{2}($

If Perimeter

top

If Δ drawn

$A = \frac{1}{2}bh$

$A(x) = \frac{1}{2} \cdot 2x\sqrt{a^2-x^2}$

$= x\sqrt{a^2-x^2}$

$A'(x) = \sqrt{a^2-x^2} + \frac{x(-2x)}{2\sqrt{a^2-x^2}}$

$\frac{a^2-x^2-x^2}{\sqrt{a^2-x^2}}$

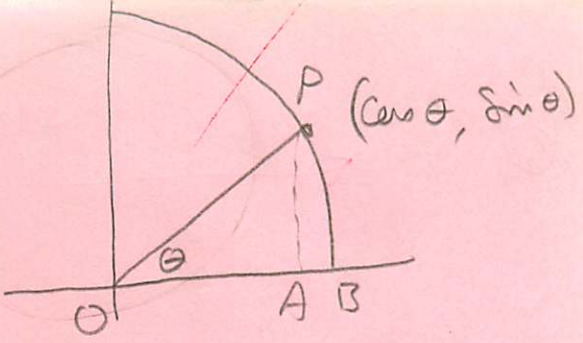
$a^2 - 2x^2 = 0$

$-2x^2 = -a^2$

$\frac{a}{\sqrt{a^2-x^2}}$

$x = \pm \frac{a}{2}$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = ?$$



$$\sin\theta = \overline{AP}$$

$$\theta = \overset{\frown}{BP}$$

what is ratio when θ is small?

$$\frac{\text{area of Circle}}{\text{Perimeter of circle (arc length of a whole circle)}} = \frac{\text{area of Sector}}{\text{arc length of Sector}}$$

$$\frac{\pi r^2}{2\pi r} = \frac{?}{\theta}$$

$$\frac{\theta r}{2} = ?$$

on unit circle $r = 1$ so

area of Sector is $\frac{\theta}{2}$.

For example we know this is $\frac{1}{4}$ of a whole circle, $\frac{\pi r^2}{4}$ $r=1$ so $\frac{\pi}{4}$

we also know arc length is $\frac{\pi}{2}$. $\frac{\pi}{2} \div 2 = \frac{\pi}{4}$ as it should.

Given $a < \frac{p}{q} < b$ and variables > 0

$a < \frac{p}{q}$ $\qquad$ $\frac{p}{q} < b$

$\frac{q}{p} < \frac{1}{a}$ $\qquad$ $\frac{1}{b} < \frac{q}{p}$

$\frac{1}{b} < \frac{q}{p} < \frac{1}{a}$

$\frac{1}{3} < \frac{7}{k} < 3$

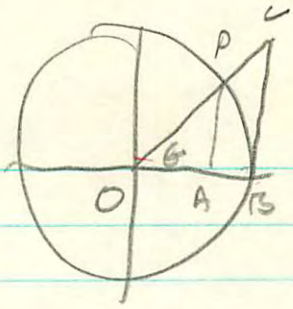
$3 > \frac{k}{7} > \frac{1}{3}$

$\frac{1}{a} < \frac{p}{q} < a$

$\frac{q}{p} < \frac{a}{1}$ $\qquad$ $\frac{1}{a} < \frac{q}{p} < \frac{a}{1}$

$x^2+y^2=1$

Sectors



We are comparing the areas of 3 items.

① Δ OAP, Sector OBP, Δ OBC

We compare $\Delta OAP \leq$ Sector $OBP \leq \Delta OBC$

$\Delta OAP \sim \Delta OBC$

$$\frac{OB}{OA} = \frac{BC}{AP}$$

$$\frac{1}{\cos\theta} = \frac{BC}{\sin\theta} \quad \therefore BC = \frac{\sin\theta}{\cos\theta}$$

Area ΔOAP ≤ area of Sector OBP ≤ area of ΔOBC

$$\frac{1}{2}\cos\theta\sin\theta \leq \frac{\theta}{2} \leq \frac{1}{2}\frac{\sin\theta}{\cos\theta}\cdot 1$$

$$\cos\theta\sin\theta \leq \theta \leq \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta \leq \frac{\theta}{\sin\theta} \leq \frac{1}{\cos\theta}$$

in quadrant I all are positive.

Now take $\lim \theta \to 0$

$\lim_{\theta\to 0} \cos\theta = 1$ ✓

$$1 \leq \lim_{\theta\to 0}\frac{\theta}{\sin\theta} \leq \frac{1}{1}$$

so

$$\frac{1}{\cos\theta} \geq \frac{\sin\theta}{\theta} \geq \cos\theta$$

$$1 \geq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \geq 1$$

ie $$1 \leq \lim_{\theta\to 0}\frac{\theta}{\sin\theta} \leq 1$$

Since the Quotient $\frac{\theta}{\sin\theta}$ is trapped between 1 & 1 we can say it is 1. * Since this ratio has value 1 then so does its reciprocal. ie

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

$$\cos x \cdot \sin^0 x$$

$$\sin x \left(-\sin x\right) + \cos x \cos x$$

$$-\sin^2 x + \cos^2 x$$

end of lecture.

Next consider $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta}$

$$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta} = \lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta(1+\cos\theta)} =$$

$$\lim_{\theta\to 0} \frac{\sin^2\theta}{\theta(1+\cos\theta)} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{1+\cos\theta}$$

$$\downarrow \qquad \downarrow$$

$$1 \cdot \frac{0}{1+1} = 0$$

Now the reason we bothered with all that was.....

Given $f(x) = \sin x$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h\to 0} \frac{\sin x\cos h + \sin h\cos x - \sin x}{h}$$

$$\lim_{h\to 0} -\sin x \frac{(1-\cos h)}{h} + \frac{\sin h\cos x}{h}$$

$$\lim_{h\to 0} -\sin x \left(\frac{1-\cos h}{h}\right) + \frac{\sin h}{h}\cos x$$

$$0 + 1 \cdot \cos x = \cos x$$

Thus $f(x) = \sin x$, $f'(x) = \cos x$,

$$(\cos x)' = -\sin x$$

Finally $d(\sin A) = \cos A\, dA$

Lecture Tuesday Day VI and VII of ISD is assigned.

Pg 1

Given $f(x) = \left(\frac{(3x^2-2)^2 (2-4x^3)^2}{\sqrt{1-x^2}} \right)^3$

Take Home But do NOT simply turn in anything. Write on paper in LANDSCAPE.

$$3\left(\int^2 \left[\sqrt{1-x^2}(2-4x^3)^2 2(3x^2-2)6x + (3x^2-2)^2 2(2-4x^3)(-12x^2) \right. \right.$$

$$\left. \left. - (3x^2-2)^2 (2-4x^3)^2 \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) \right] \right)$$

$$1-x^2$$

That's where you start. I will check for proper items in the proper place. Deviations will be considered wrong.

$f(x) = \cos x - x \qquad -2\pi \le x < 2\pi$

$f'(x) = -\sin x - 1 \overset{SET}{=} 0$

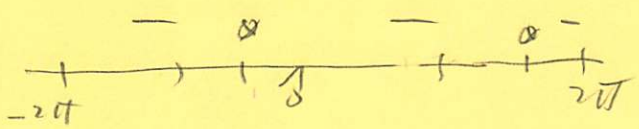
$\sin x = -1$

$x = \left\{-\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

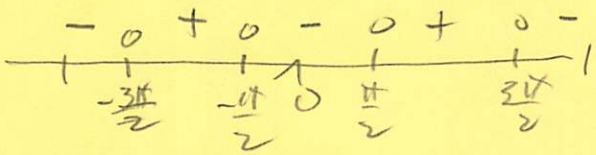
$f''(x) = -\cos x \overset{SET}{=} 0$

$x = \left\{-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

f'



f''



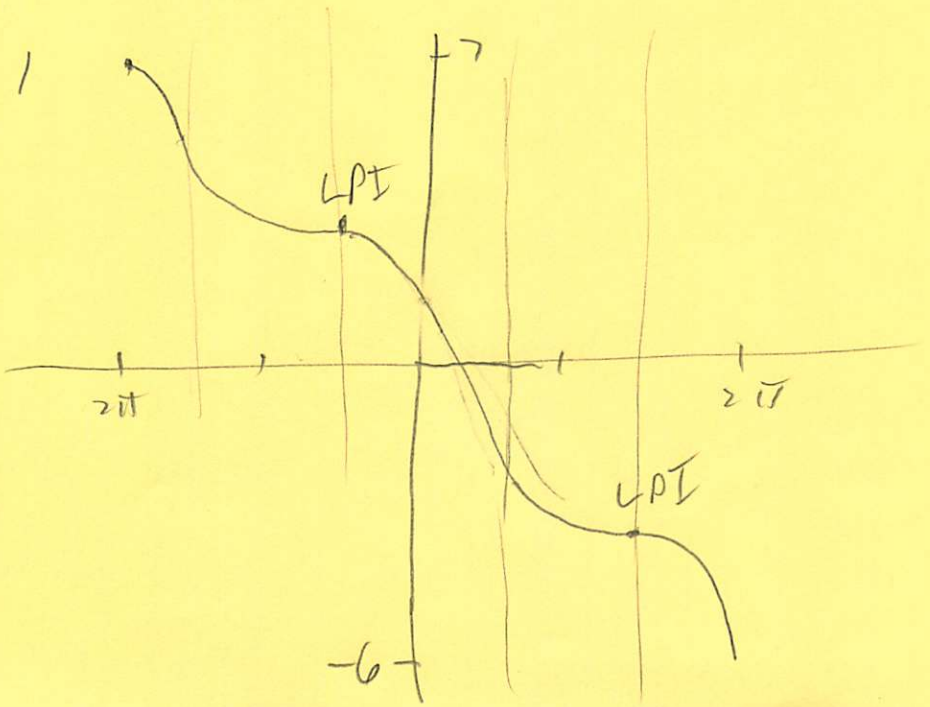
Notice There are Non-participating Points because $\sin x \le 1$ so $-\sin x - 1$ is Never > 0.

$f(-2\pi) = 1 + 2\pi \approx 7$ abs max ENDPT dec fn'ly.

$f(2\pi) = 1 - 2\pi \approx -6$ abs min ENDPT dec to it

LPI at $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, -\frac{3\pi}{2}\right)$

$f(0) = 1$



VI

D) $f(x) = \sin\left(\frac{x^2}{1+x^2}\right)$ $\quad -2\pi \le x \le 2\pi$

$$f'(x) = \cos\left(\frac{x^2}{1+x^2}\right)\left[\frac{(1+x^2)(2x) - x^2(2x)}{(1+x^2)^2}\right]$$

$$= \cos\left(\frac{x^2}{1+x^2}\right)\left[\frac{2x}{(1+x^2)^2}\right] \overset{\text{set}}{=} 0$$

f' : $\quad -$ $\quad 0$ $\quad +$ $\quad$ (0 is a minimum)

f'' is too ugly.

$\frac{x^2}{1+x^2} \ge 0 \ \forall x$

$\frac{x^2}{1+x^2} \overset{?}{=} \frac{\pi}{2}$

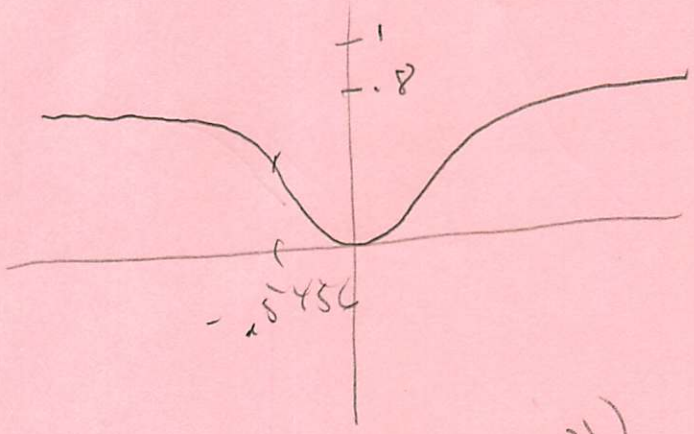
$2x^2 = \pi + \pi x^2$

$x^2 = \frac{\pi}{2-\pi} < 0$

reject $x^2 < 0$

$\frac{x^2}{1+x^2} < \frac{\pi}{2}$

so $\cos\left(\frac{x^2}{1+x^2}\right) > 0$



graphed f'' to see that concavity does change

$Y_1 = \sin(x^2/(1+x^2))$

$Y_2 = nDeriv(Y_1, X, X)$

$Y_3 = nDeriv(Y_2, X, X)$

INDustrial Strength Sin

VI

c) $f(x) = \dfrac{\cos x}{1+\sin x}$

$x \neq \left\{-\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ are asymptotes

$D_f \quad x \neq \left\{-\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

$$f'(x) = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$1+\sin x$ Never negative.

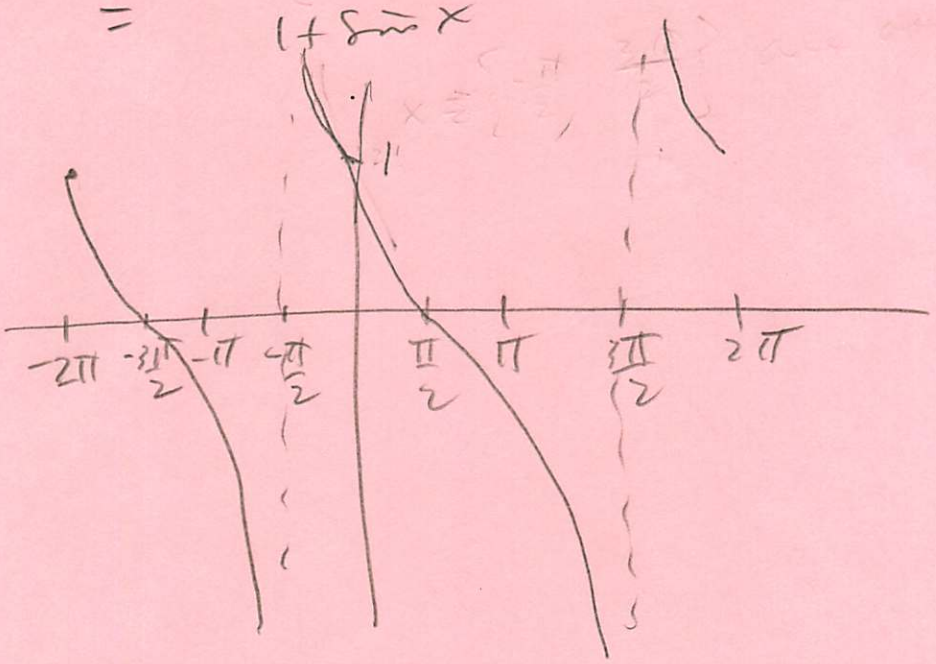
$$= -\sin x - \sin^2 x - \cos^2 x$$

$$= -\sin x - \sin^2 x - (1-\sin^2 x)$$

$$= -\sin x - \sin^2 x - 1 + \sin^2 x$$

$$= \frac{-(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{-1}{1+\sin x} \leq 0 \quad \forall x$$



(Indus) Real Stuff
Dan Anlch4

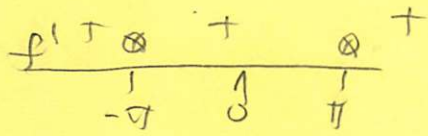
U1

a) $f(x) = \sin x + x$

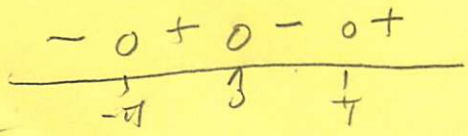
$f'(x) = \cos x + 1 \overset{\text{Set}}{=} 0$

$\cos x = -1$

$x \in \{-\pi, \pi\}$



$f''(x) = -\sin x$

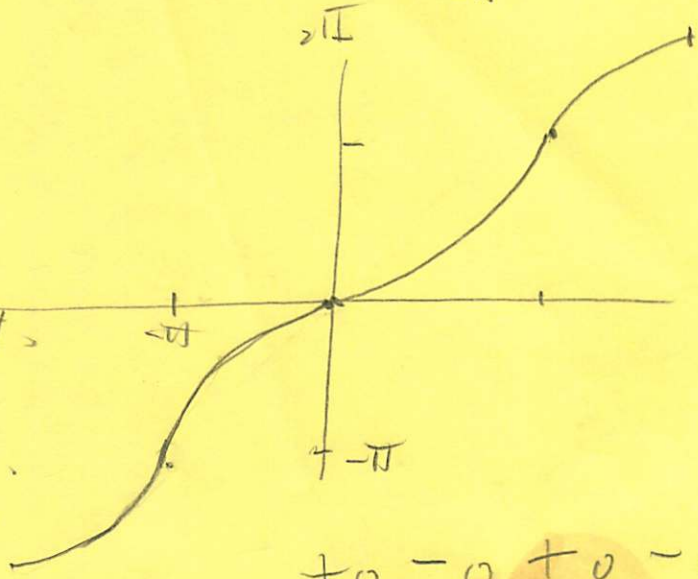


$(-\pi, -\pi)$ PI

(π, π) PI

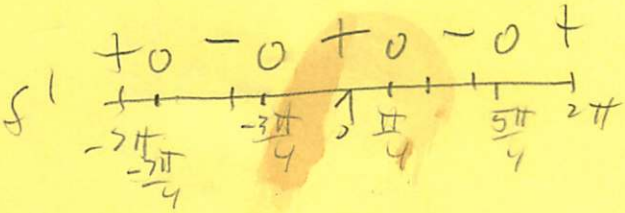
min $(2\pi, -2\pi)$
ENDPT inc from it.

max $(2\pi, 2\pi)$
ENDPT inc to it.



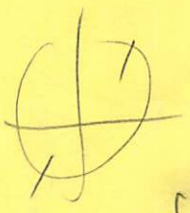
b) $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x \overset{\text{Set}}{=} 0$

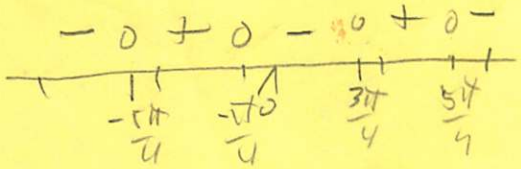


$\tan y = 1$

$x \in \left\{-\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$

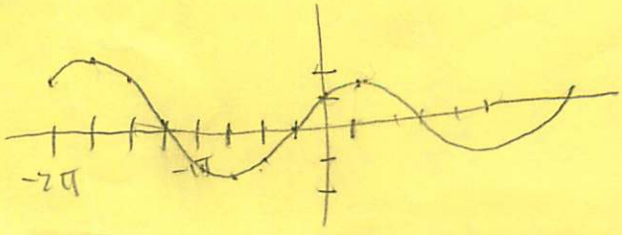


$f'' = -\sin x - \cos x = 0$



$1 - \tan x$

$\frac{\sin x}{\cos x} = -1$



Lookalike $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

IMAginethat! :)

15 minutes.

Given 4, find the equation of the tangent line at the given point.

(1) $x^2 + 2x^3y^2 + y^4 = 4$ at $(1,1)$

$$2x + 6x^2y^2 + 4x^3y\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 0$$

$$(4x^3y + 4y^3)\frac{dy}{dx} = -2x(1+3xy^2)$$

$$\frac{dy}{dx} = \frac{-2x(1+3xy^2)}{4y(x^3+y^2)}\Bigg|_{(1,1)}$$

$$\frac{-2(4)}{4(2)} = -1 = m \quad (1)$$

$y - 1 = -(x-1)$ (1) (1)

$y = -x + 2$

(2) $y = \dfrac{(x^2+2)(x-3)}{x+4}$

$$\frac{dy}{dx} = \frac{(x+4)\left[(x-3)2x + x^2+2\right] - (x^2+2)(x-3)}{(x+4)^2}$$

$$\frac{(x+4)\left[2x^2-6x+x^2+2\right] - x^3-3x^2+2x-6}{(x+4)^2}$$

$$\frac{(x+4)(3x^2-6x+2) - (x^2+2)(x-3)}{(x+4)^2}\Bigg|_{x=0} =$$

$$\frac{4(2) - (2(-3))}{16}$$

$$\frac{8+6}{16} \quad \frac{14}{16} = \frac{7}{8} = m \quad (1)$$

$$y\Big|_{x=0} = \frac{2(-3)}{x_2} = -\frac{3}{2} \quad (1)$$

$m = \frac{7}{8}$ $\left(0, -\frac{3}{2}\right)$

$y + \frac{3}{2} = \frac{7}{8}x$

$y = \frac{7}{8}x - \frac{3}{2}$ (1)

56 +
4 ✓
3 0
2 −
0 1 =

No Calculator

AnL ch 4m

Quiz #1

Given y: find equation of tangent line at given pt.
Given y find $\frac{dy}{dx}$ evaluate at the given pt.

① $x^2 + 2x^3y^2 + y^4 = 4$ (u, v)

$2x + 6x^2y^2 + 4x^3y\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 0$

$2 + 6 + 4\frac{dy}{dx} + 4\frac{dy}{dx} = 0$

at x (1, 1)

$8\frac{dy}{dx} = -8$ 1pt

$\frac{dy}{dx} = -1 = m$

$y - 1 = -1(x - 1)$

$= -x + 1$

$y = -x + 2$ 1pt

② $y = \frac{(x^2+2)(x-3)}{x+4}$ (u, v)

$\frac{(x+4)[(x-3)2x + (x^2+2)] - (x^2+2)(x-3)}{(x+4)^2}$

at x = 0

$\frac{4[0+2] + 6}{16}$

$\frac{8+6}{16}$

$\frac{7}{8}$ 1pt

$2(-3)$

$-4 - 2$

-6

$\frac{-3}{2}$ 1pt $(0, -\frac{3}{2})$

$y + \frac{3}{2} = \frac{7}{8}(x)$

$y = \frac{7}{8}x - \frac{3}{2}$

pts

4 +

3 ✓

2 0

1 −

0 =

15 minutes

I "Write equations of tangent lines to given when $x=1$."

$x y^2 - x^3 y = 6$

$x=1$ $\quad y^2 - y - 6 = 0$

$(y-3)(y+2)=0$

Points are $(1, 3)$ and $(1,-2)$ ① ②

Need Slope

$$y^2 + 2xy\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}\Bigg| = \frac{3\cdot1\cdot3-9}{6-27} \quad ③$$

$$0 = m$$

$(1,3)$

$$\Big|_{(1,-2)} = \frac{3\cdot1(-2)-4}{2\cdot(1)(-2)-1}$$

$$\frac{-10}{-5} = 2 \quad ④$$

$y = 3$ ⑤ $\qquad y+2 = 2(x-1)$

$y = 2x - 4$ ⑥

10 minutes

11, 12 +

9, 10 V

4, 7, 8 O

3 4 5 —

0 -0, 1, 2 =

Quiz

II Determine the coordinates of all significant points. Justify. Sketch (which includes all 6 points)

$f(x) = 3x^4 - 4x^3 \quad x \in [-1, 2]$

$x^3(3x-4)$

① Non partic pt.

$f'(x) = 12x^3 - 12x^2$

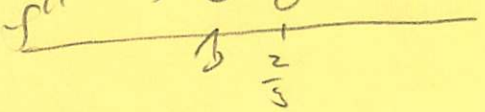
$12x^2(x-1)$

Both # line ①

$\frac{8}{27}, (-1)$

$f''(x) = 36x^2 - 24x$

$12x(3x-2)$



$(-1, 7)$ max endpt dec fm it.

$(0, 0)$ LPI

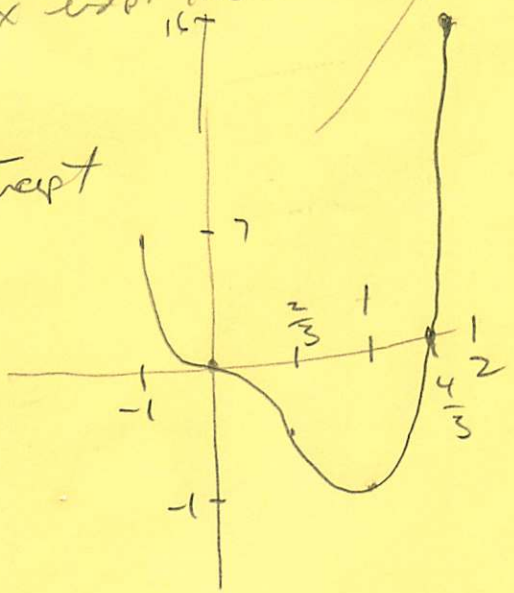
$(\frac{2}{3}, \frac{-16}{27})$ PI

$(1, -1)$ min f' test

$(2, 16]$ max endpt inc to it.

$(\frac{4}{3}, 0)$ x intercept

(3) in pairs must be correct and properly justified



① must have pts indicated